

Cloning Quantum Channels

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QuantumBasel and University of Basel

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Talk outline

01

Quantum states

Cloning

Replicating

02

Quantum channels

Higher-order processes

Cloning:

- Trash-and-replace channel
- Unitaries
- Beyond unitaries

Results: no-gos and gos, cloning strategies

03

Summary and outlook

What we can and cannot clone

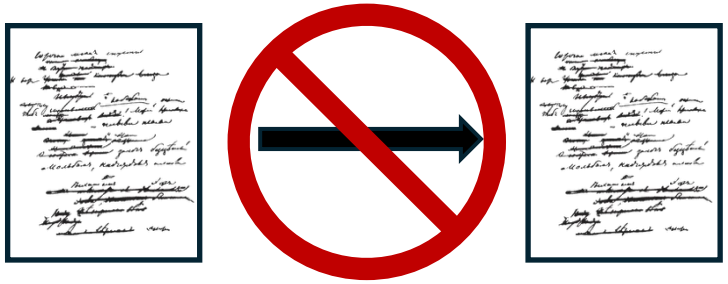
Open questions and next steps

Puzzle

Cloning quantum states

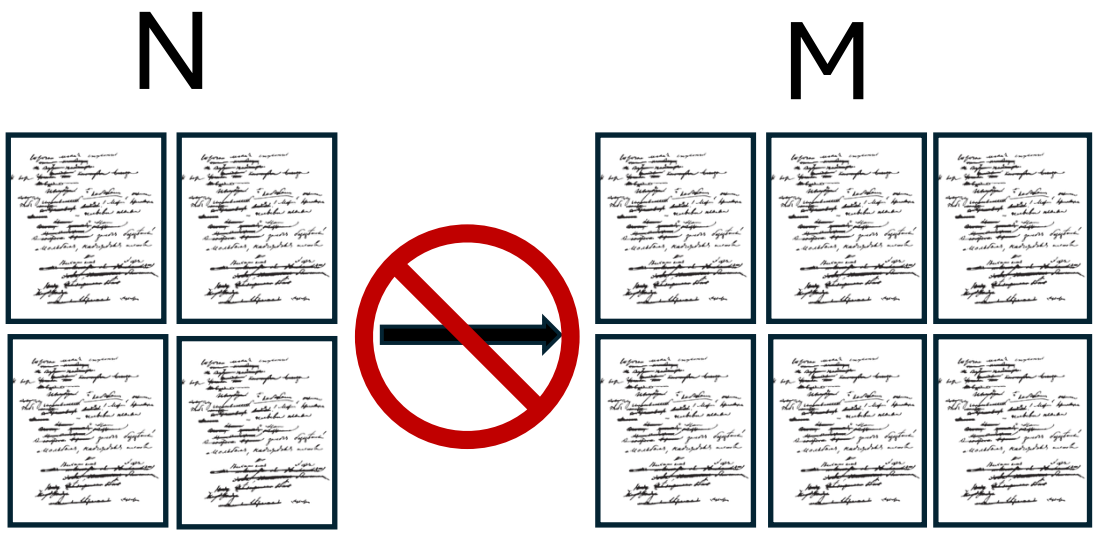
Cloning quantum states





Cloning quantum states

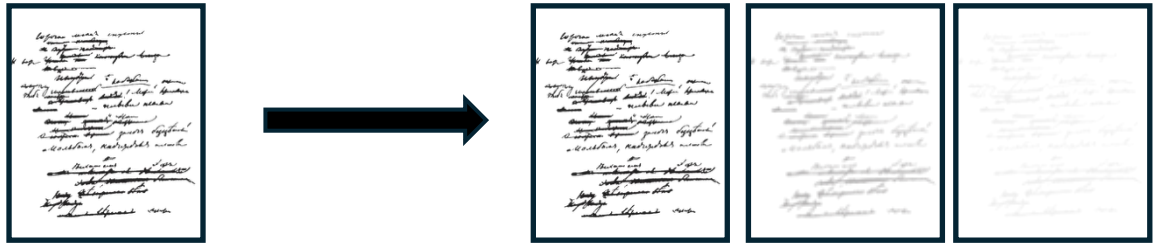




Cloning quantum states

Approximately





Cloning quantum states

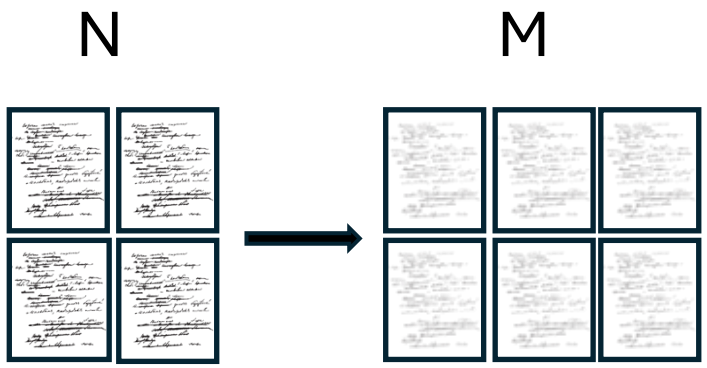
Approximately

1. V. Bužek and M. Hillery. "Quantum Copying: Beyond the No-Cloning Theorem". *Phys. Rev. A* 54, 1844–1852 (1996). doi:10.1103/PhysRevA.54.1844
2. N. Gisin and S. Massar. "Optimal Quantum Cloning Machines". *Phys. Rev. Lett.* 79, 2153–2156 (1997). doi:10.1103/PhysRevLett.79.2153
3. R. F. Werner. "Optimal Cloning of Pure States". *Phys. Rev. A* 58, 1827–1832 (1998). doi:10.1103/PhysRevA.58.1827
4. D. Bruß, A. Ekert, and C. Macchiavello. "Optimal Universal Quantum Cloning and State Estimation". *Phys. Rev. Lett.* 81, 2598–2601 (1998). doi:10.1103/PhysRevLett.81.2598
5. L.-M. Duan and G.-C. Guo. "Probabilistic Cloning and Identification of Linearly Independent Quantum States". *Phys. Rev. Lett.* 80, 4999–5002 (1998). doi:10.1103/PhysRevLett.80.4999
6. A. K. Pati. "Quantum Superposition of Multiple Clones and the Novel Cloning Machine". *Phys. Rev. Lett.* 83, 2849–2852 (1999). doi:10.1103/PhysRevLett.83.2849
7. A. Chefles and S. M. Barnett. "Strategies and Networks for State-Dependent Quantum Cloning". *Phys. Rev. A* 60, 136–144 (1999). doi:10.1103/PhysRevA.60.136
8. D. Bruß et al. "Phase-Covariant Quantum Cloning". *Phys. Rev. A* 62, 012302 (2000). doi:10.1103/PhysRevA.62.012302
9. G. M. D'Ariano and C. Macchiavello. "Optimal Phase-Covariant Cloning for Qubits and Qutrits". *Phys. Rev. A* 67, 042306 (2003). doi:10.1103/PhysRevA.67.042306

Approximate cloning

Given N copies of ρ produce an output *approximating* $M > N$ copies.

$$\rho^{\otimes N} \rightarrow \mathcal{P}\left[\rho^{\otimes N}\right] \approx \rho^{\otimes \mathrm{M}}$$



Cloning quantum states

Quality of the copies

Figure of merit: the canonical quantity in quantum information is the fidelity

$$F(\rho,\sigma) := \mathrm{tr}\,|\sqrt{\rho}\sqrt{\sigma}| = \mathrm{tr}\,\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}}$$

***Worst case* cloning** with respect to the figure of merit.

$$\mathsf{F}^{\mathcal{S}}(N,M) := \max_{\mathcal{P}} \inf_{\rho\in\mathcal{S}} F\left(\mathcal{P}[\rho^{\otimes N}], \rho^{\otimes M}\right)$$

per-copy fidelity



global fidelity



Cloning quantum states

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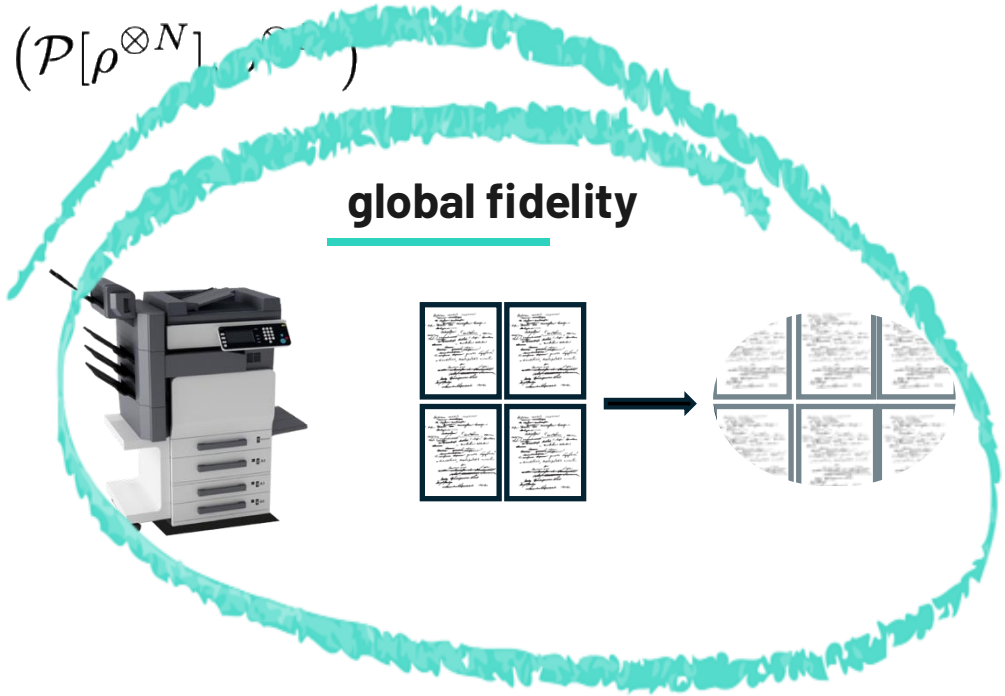
$$\mathsf{F}^{\mathcal{S}}(N,M) := \max_{\mathcal{P}} \inf_{\rho\in\mathcal{S}} F\left(\mathcal{P}[\rho^{\otimes N}], \quad\right)$$

CONTINUOUS SETS ONLY !

per-copy fidelity



global fidelity



Replicating quantum states

Good quality cloning

Replication = high-quality cloning

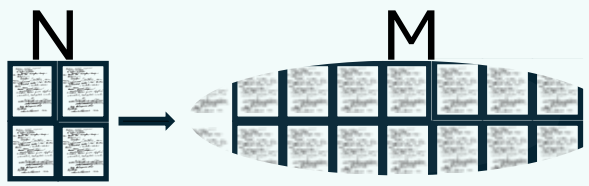
Given N copies of ρ produce an output approximating $M>N$ copies with a bounded error $\epsilon \ll 1$

$$\mathsf{M}^{\mathcal{S}}(N,\epsilon) = \max_{\mathcal{P}} \quad M$$
$$\text{s.t.} \quad \inf_{\rho\in\mathcal{S}} F^{\mathcal{S}}\left(\mathcal{P}[\rho^{\otimes N}], \rho^{\otimes M}\right) \geq 1-\epsilon\,.$$

Replication rate: the scaling of M in the large N limit.

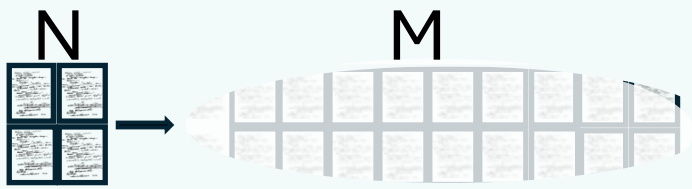
Super-replication: high-quality quadratic cloning, can't get better than that $M \leq \mathcal{O}(N^2)$

$$\mathsf{R}^{\mathcal{S}}(\epsilon) = \sup\, r \quad \text{s.t.} \quad \lim_{N\to\infty} \frac{\mathsf{M}^{\mathcal{S}}(N,\epsilon)}{N^{\,r}} > 0.$$



super-replication

$M \sim \mathcal{O}(N^2)$



post super-replication

$M > \mathcal{O}(N^2)$

Replicating quantum states

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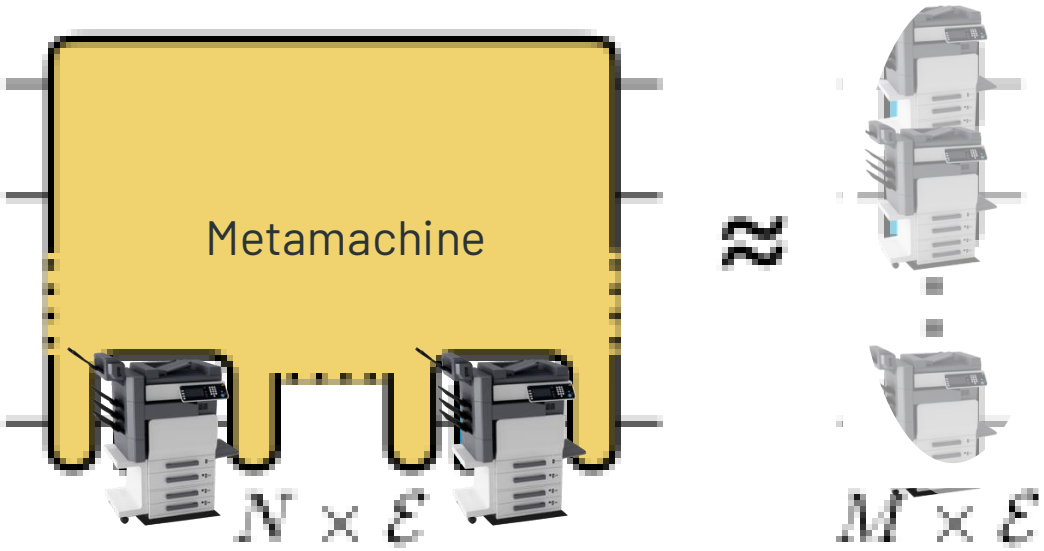
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- Arbitrary quantum states achieve a linear rate replication rate (deterministic protocol)[9]
- Clock states $\{|\psi_t\rangle = e^{-itH}|\psi\rangle\}$ achieve a quadratic replication rate (probabilistic)[10].

9. G. M. D'Ariano and C. Macchiavello. "Optimal Phase-Covariant Cloning for Qubits and Qutrits". *Phys. Rev. A* 67, 042306 (2003). doi:10.1103/PhysRevA.67.042306
10. G. Chiribella, Y. Yang, and A. C.-C. Yao. "Quantum Replication at the Heisenberg Limit". *Nat. Commun.* 4, 2915 (2013). doi:10.1038/ncomms3915

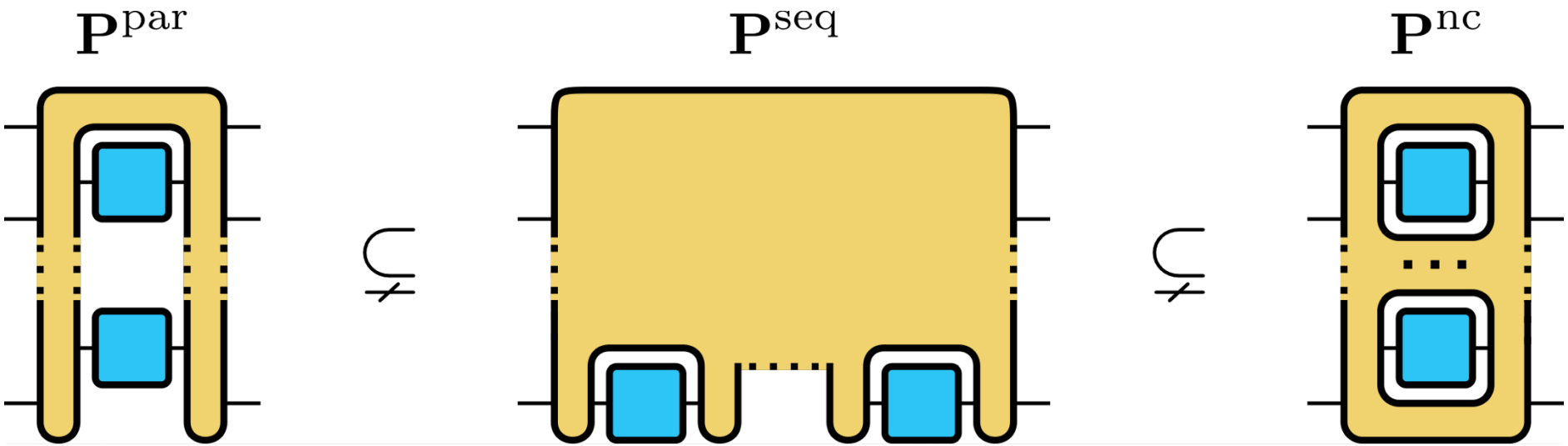
Cloning quantum channels

Cloning quantum channels



Higher order processes

Types of metamachine



In the **CJ isomorphism** each process is characterised by necessary and sufficient conditions; in each case the linear constraints are more restrictive.

Higher order processes

Types of metamachine

TPP

Trace preserving preserving

Trace preserving maps are mapped to trace preserving maps.

CPP

Completely positive preserving

Completely positive maps are mapped to to completely positive maps.

CCPP

Completely completely positive preserving

Map remains CPP, i.e. sends CP maps to CP maps even when applied locally to multipartite transformations.

In the **CJ isomorphism** each process is characterised by necessary and sufficient conditions; in each case the linear constraints are more restrictive.

Figure of merit

Channel closeness

Recall for states

Figure of merit for the closeness of quantum states was the fidelity $F(\rho,\sigma) := \mathrm{tr}\,|\sqrt{\rho}\sqrt{\sigma}| = \mathrm{tr}\,\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}}$

Optimal cloning fidelity:

$$\mathsf{F}^{\mathcal{S}}(N,M) := \max_{\mathcal{P}} \inf_{\rho\in\mathcal{S}} F\left(\mathcal{P}[\rho^{\otimes N}], \rho^{\otimes M}\right)$$

For channels

Average fidelity (only works if at least one channel unitary)

$$\overline{F}(\mathcal{U},\mathcal{V}) := \langle\psi| \int d\psi\, V^\dagger(U|\psi\rangle\langle\psi|U^\dagger)V|\psi\rangle = \frac{dF_{\mathrm{CJ}}+1}{d+1}$$

$$F_{\mathrm{CJ}}(\mathcal{A},\mathcal{B}) := F((\mathrm{id}\otimes\mathcal{A})[\Phi^+], (\mathrm{id}\otimes\mathcal{B})[\Phi^+])$$

11. M. Horodecki, P. Horodecki, and R. Horodecki. "General Teleportation Channel, Singlet Fraction, and Quasidistillation". *Phys. Rev. A* 60, 1888–1898 (1999). doi:10.1103/PhysRevA.60.1888
12. M. A. Nielsen. "A Simple Formula for the Average Gate Fidelity of a Quantum Dynamical Operation". *Phys. Lett. A* 303, 249–252 (2002). doi:10.1016/S0375-9601(02)01272-0

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where $\Phi^+ := \frac{1}{d}\sum_{ij}|ii\rangle\langle jj|$ Is the max ent. state

Diamond fidelity

$$F_{\diamond}(\mathcal{A},\mathcal{B}) := \min_{\Psi} F\big((\mathrm{id}\otimes\mathcal{A})[\Psi], (\mathrm{id}\otimes\mathcal{B})[\Psi]]\big)$$

where min is over all pure states.

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Fidelity is not a proper distance. Easy fix: take inverse cosine

$$D_{\bullet} := \arccos F_{\bullet} \in \left[0, \tfrac{\pi}{2}\right].$$

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Fidelity is not a proper distance. Easy fix: take inverse cosine

$$D_{\bullet} := \arccos F_{\bullet} \in [0, \tfrac{\pi}{2}].$$

Optimal cloning distance. Figure of merit (closeness) between the target and approximation.

$$\mathsf{D}^{\mathcal{C}}_{\bullet}(N,M) := \min_{\mathbf{P}} \sup_{\mathcal{E}\in\mathcal{C}} D_{\bullet}(\mathbf{P}[\mathcal{E}^{\otimes N}], \mathcal{E}^{\otimes M})$$

Cloning quantum states = cloning the T&R channel

T&R channel is a machine, which for any input state trashes it and replaces it with some other fixed state. Like a faulty photocopier, that regardless of the input, always prints the Girton College logo.



Cloning states is equivalent to cloning the T&R channel, i.e, cloning the faulty photocopier.

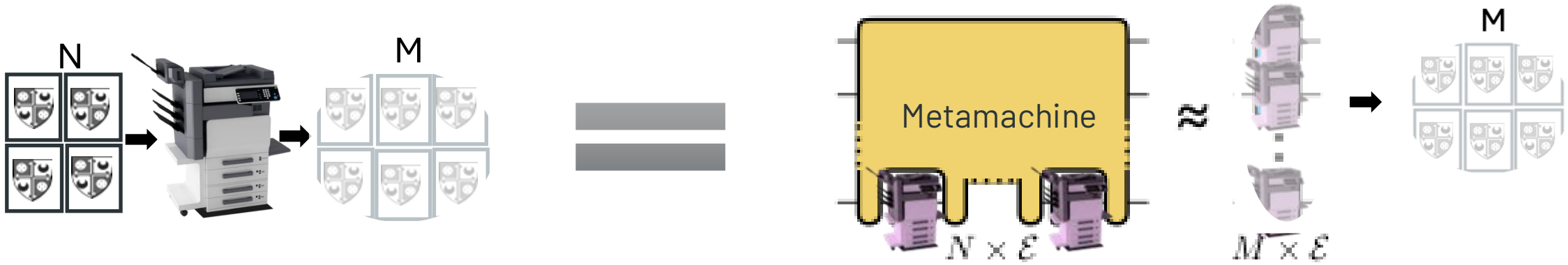


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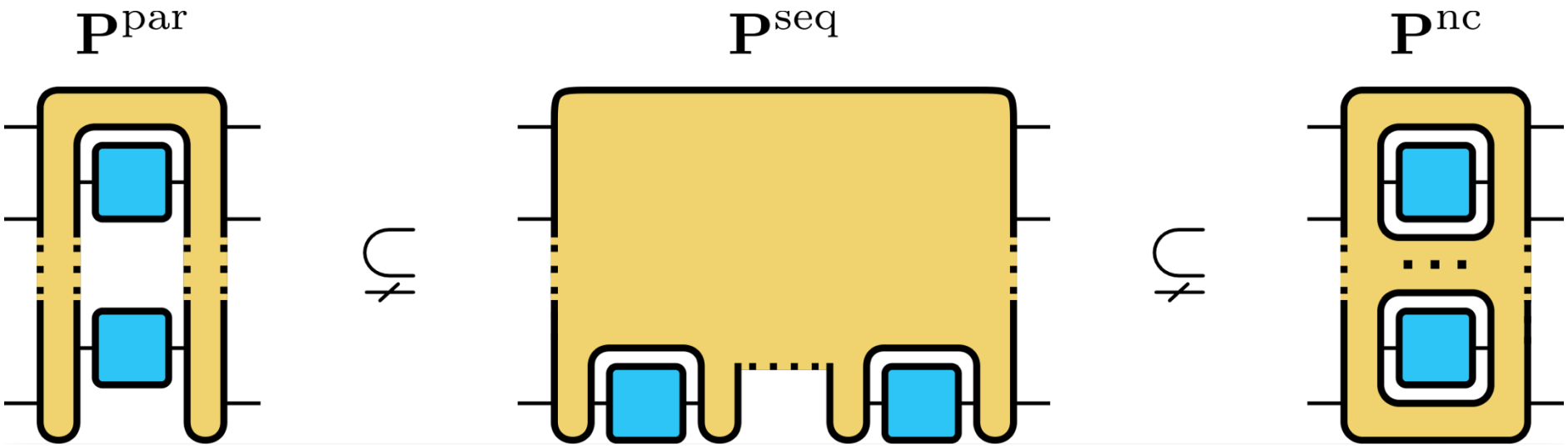


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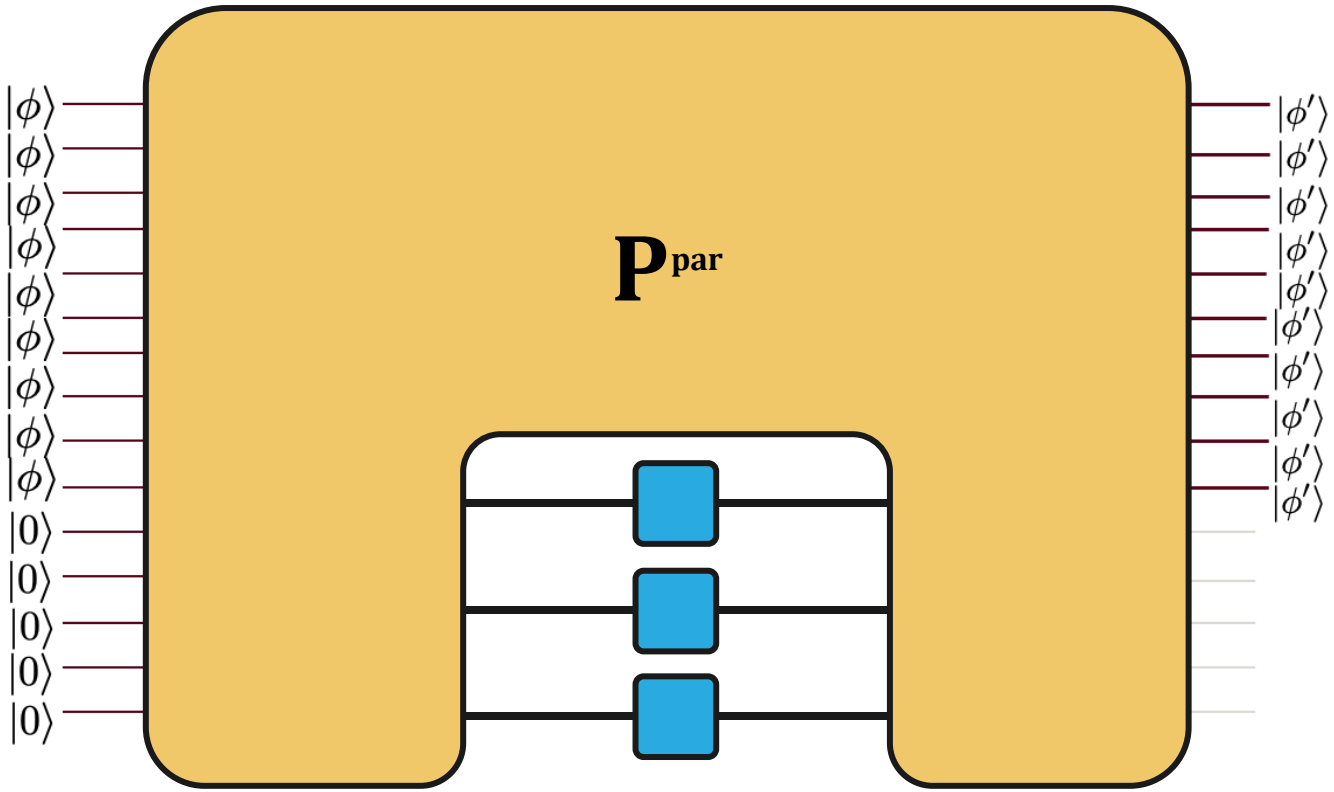
Universal framework

What other channels can we clone?



Cloning unitaries

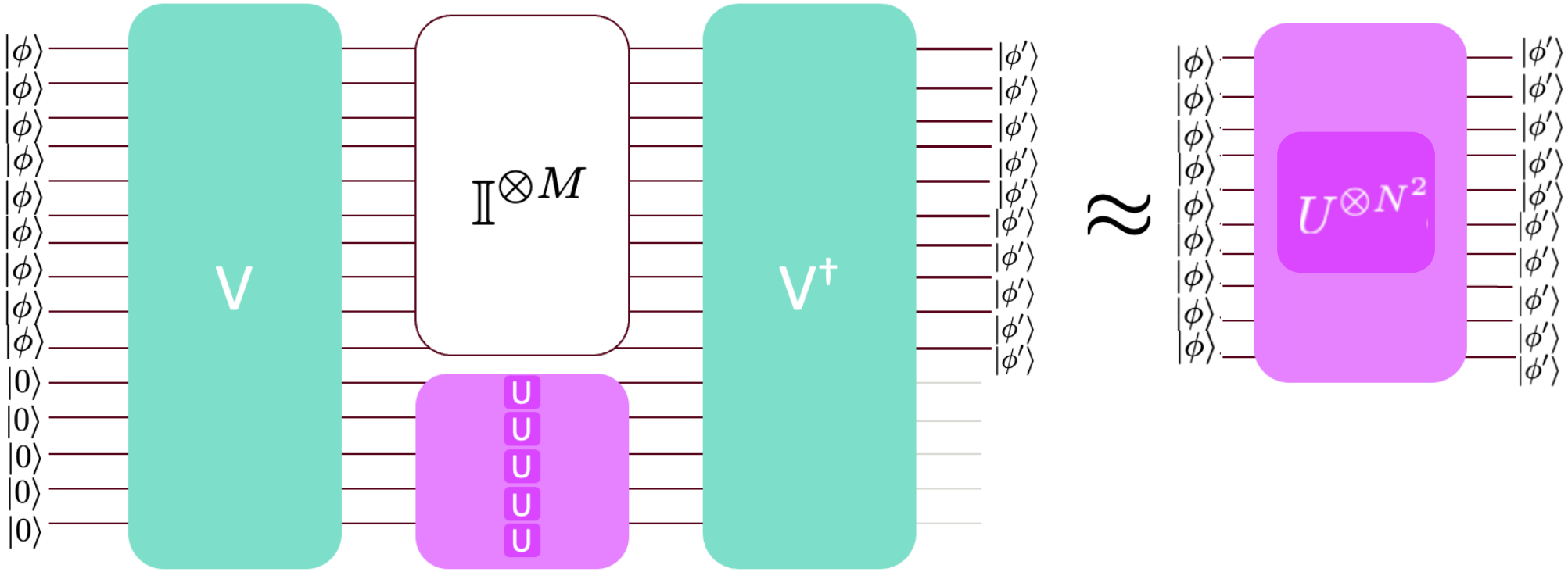
Unitary gates can be super-replicated (high-quality quadratic cloning)



13. W. Dür, P. Sekatski, and M. Skotiniotis. "Deterministic Superreplication of One-Parameter Unitary Transformations". *Phys. Rev. Lett.* 114, 120503 (2015). doi:10.1103/PhysRevLett.114.120503
14. G. Chiribella, Y. Yang, and C. Huang. "Universal Superreplication of Unitary Gates". *Phys. Rev. Lett.* 114, 120504 (2015). doi:10.1103/PhysRevLett.114.120504

Cloning unitaries

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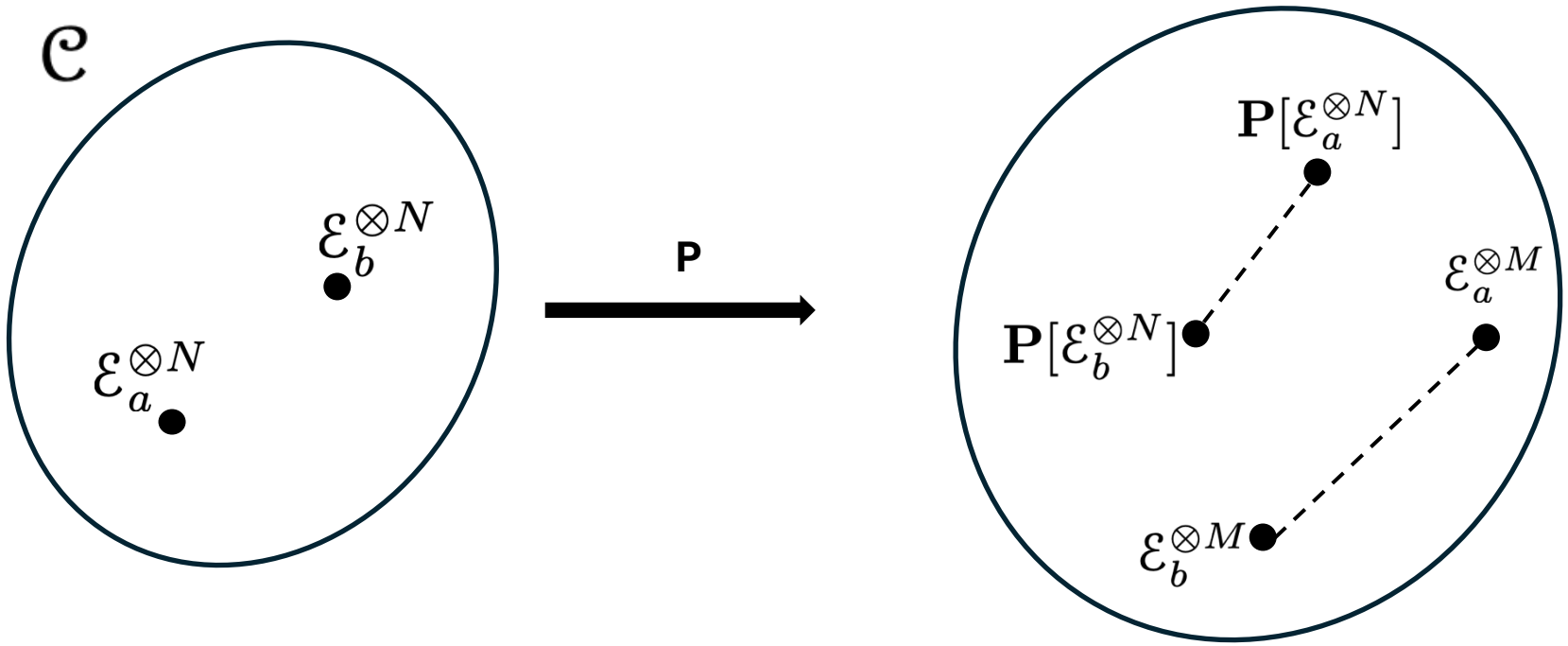
Beyond unitary cloning

Cloning general CPTP maps

Lower bound on the optimal distance

$$x \leq \mathsf{D}_\bullet^{\mathcal{C}}(N,M) := \min_{\mathbf{P}} \sup_{\mathcal{E}\in\mathcal{C}} D_\bullet(\mathbf{P}[\mathcal{E}^{\otimes N}], \mathcal{E}^{\otimes M})$$

Lower bound on the optimal distance

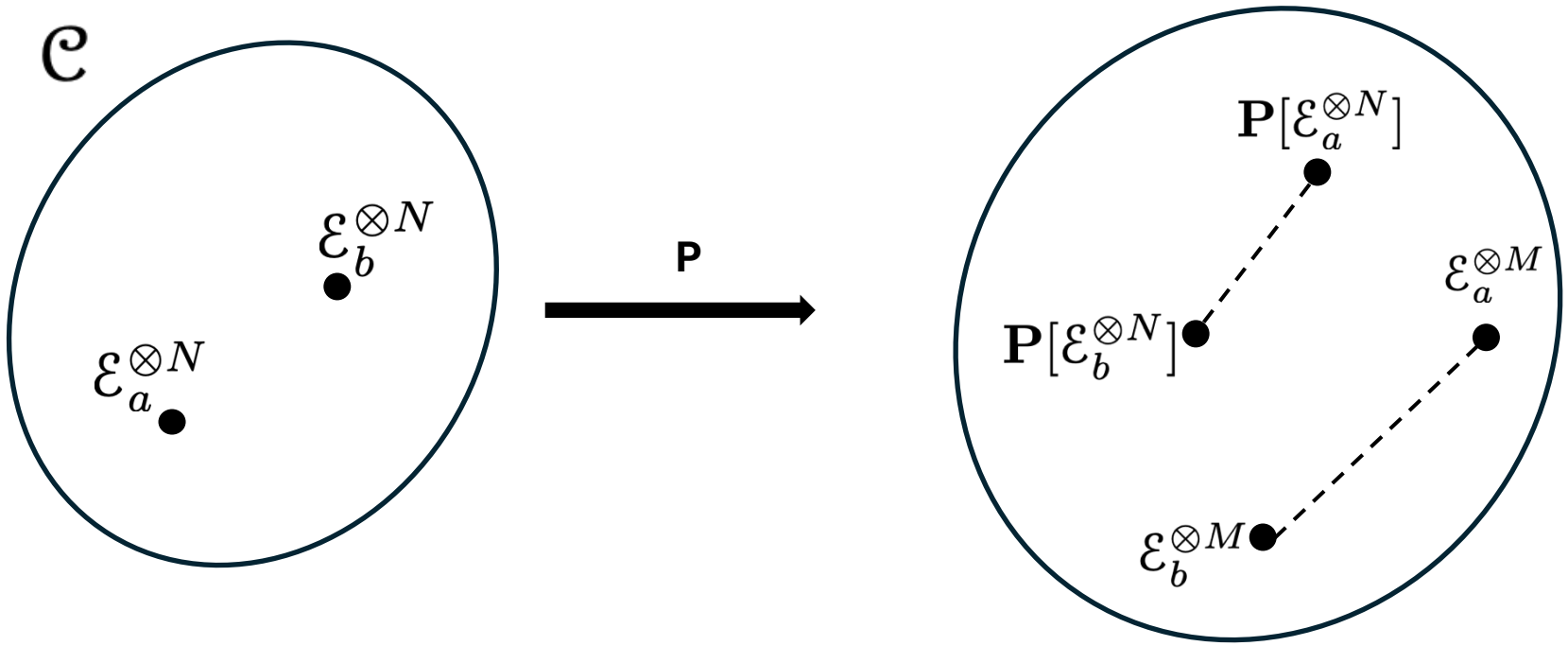


In the geometric picture there are some other distances to consider. Use these with triangle inequality to write an expression for the optimal lower bound.

$$\mathsf{D} \geq \frac{1}{2}(D(\mathcal{E}_a^{\otimes M}, \mathcal{E}_b^{\otimes M}) - D(\mathbf{P}[\mathcal{E}_a^{\otimes N}], \mathbf{P}[\mathcal{E}_b^{\otimes N}])$$

Some mathematical manipulations later…

Lower bound on the optimal distance



Connect with the QFI, then bound it with a function. Under any **causally ordered** state-producing process this was bounded by Kurdziałek et al.[15]

To use this bound we want a and b **very close but not equal**, b = a + δ, where δ = δ (M) then take the limit of M⟶ ∞.

$$\mathrm{D}_{\bullet} \geq \frac{1}{2}\left(D_{\bullet}(\mathcal{E}_a^{\otimes M}, \mathcal{E}_b^{\otimes M}) - \int_a^b \sqrt{f_N(\mathcal{E}_x)}\, dx\right)$$

$$f_N(\mathcal{E}_x) := \min\ N\sqrt{\|\alpha(x)\|}\left((N-1)\|\beta(x)\| + \sqrt{\|\alpha(x)\|}\right)$$

$$\alpha(x) := \sum_n \dot{K}_n^\dagger(x)\dot{K}_n(x) \qquad \beta(x) := \sum_n \dot{K}_n^\dagger(x)K_n(x)\,,$$

15. S. Kurdziałek et al. "Using Adaptiveness and Causal Superpositions Against Noise in Quantum Metrology". *Phys. Rev. Lett.* 131, 090801 (2023). doi:10.1103/PhysRevLett.131.090801

Necessary conditions for super-replication

It all boils down to β

$$\alpha(x) := \sum_n \dot{K}_n^\dagger(x)\dot{K}_n(x)$$
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- If there exists a Kraus decomposition of $\mathcal{E}_x$ such that $\beta(x) = 0$ then the optimal replication rate is **linear.**
- If there does not exist a Kraus decomposition such that $\beta(x) = 0$ then it **may be possible** to super-replicate the channel set.

Replication = high-quality cloning

Super-replication no goes

When the process is causally ordered

No super-replication of classical noise channels

$\mathcal{C}$ is a set of channels $\{\mathcal{E}_x \,|\, x \in [0,\delta)\} \subset \mathcal{C}$ with $\mathcal{E}_x[\,\cdot\,] = \sum_k K_k(x) \cdot K_k^\dagger(x)$

$$K_k(x) = a_k(x) G_k \qquad a_k(x) \in \mathbb{R},\ G_k \in L(\mathcal{H})\,,$$

Including depolarising, bit-flip, stochastic, etc.

No super-replication of the amplitude damping map

The derivatives of the Kraus operators are $\beta(\gamma) = \dot{K}_0^\dagger(\gamma) K_0(\gamma) + \dot{K}_1^\dagger(\gamma) K_1(\gamma) = 0$ which rules out the possibility of super-linear replication.

Kraus operators are:

$$K_0(\gamma) = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}$$

$$K_1(\gamma) = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}.$$

No super-replication of the full set of noisy quantum gates

Let $\mathcal{N}[\,\cdot\,] = \sum_k L_k \cdot L_k^\dagger$ be a non-unitary channel. The set of noisy unitary gates

$$\mathcal{C} = \{\mathcal{N} \circ \mathcal{U} | \mathcal{U} \in \mathrm{SU}(d)\}$$

Implication: super-replication of SU(d) has zero robustness.[14]

Open question: could non-causally ordered processes lead to super-replication in any of these cases?

14. G. Chiribella, Y. Yang, and C. Huang. "Universal Superreplication of Unitary Gates". *Phys. Rev. Lett.* 114, 120504 (2015). doi:10.1103/PhysRevLett.114.120504

Super-replication no goes

Fundamental Limits of Quantum Metrology Beyond Fixed Causal Order

Wenjie Wei[1], Yutong Li[1], and Shengshi Pang[1,2]*
[1] *School of Physics, Sun Yat-sen University, Guangzhou, Guangdong 510275, China and*
[2] *Hefei National Laboratory, University of Science and Technology of China, Hefei 230088, China*
(Dated: September 7, 2026)

Quantum metrology with indefinite causal order (ICO) has attracted intense interest due to its potential to surpass the limitations of conventional fixed-order strategies. A key open question is whether ICO can fundamentally enhance asymptotic precision scaling. In this work, we bridge this gap for the estimation of a single parameter encoded in N identical uses of a finite-dimensional quantum channel. We first establish a universal Heisenberg-scaling upper bound for the full general ICO process-matrix class and show that for unitary channels its optimal quantum Fisher information (QFI) coincides exactly with that of parallel strategies. For noisy channels, a structurally refined bound shows that channels restricted to the standard quantum limit (SQL) under parallel strategies remain SQL-limited under general ICO strategies. Most significantly, an asymptotically tight (AT) bound is derived to close the remaining possibility of an asymptotic ICO advantage by showing that general ICO and optimal parallel strategies have exactly the same leading QFI coefficient in both the SQL and Heisenberg regimes. For the operationally motivated class of quantum circuits with quantum control of causal order, we further obtain an iterative constraint on finite-query precision whose asymptotic limit agrees with that of the AT bound. Our results clarify the ultimate role of indefinite causality as a metrological resource for quantum channel estimation.

Sep 2026

+ ChatGPT

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probably not

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$$\beta(x) := \sum_n \dot{K}_n^\dagger(x)K_n(x)\,,$$

- If there exists a Kraus decomposition of $\mathcal{E}_x$ such that $\beta(x) = 0$ then the optimal replication rate is **linear.**
- If there does not exist a Kraus decomposition such that $\beta(x) = 0$ then it **may be possible** to super-replicate the channel set.

- **If β is not zero, what then?**

Necessary conditions for super-replication

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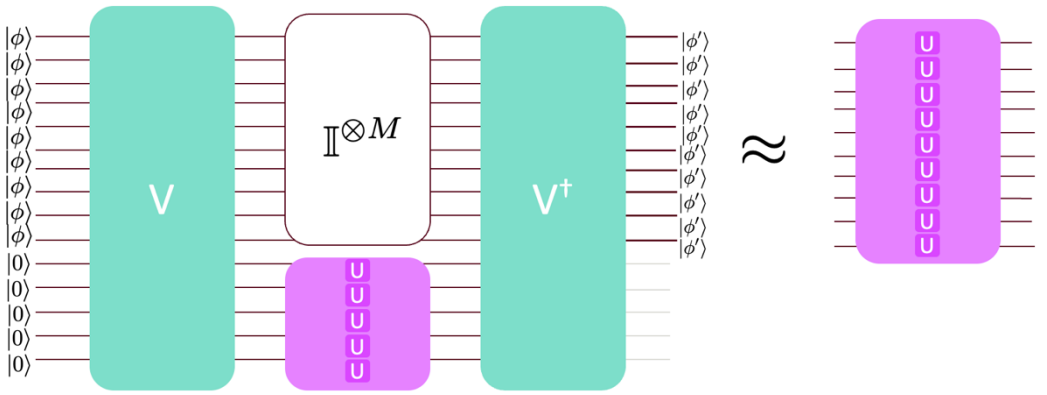
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Channel cloning strategies

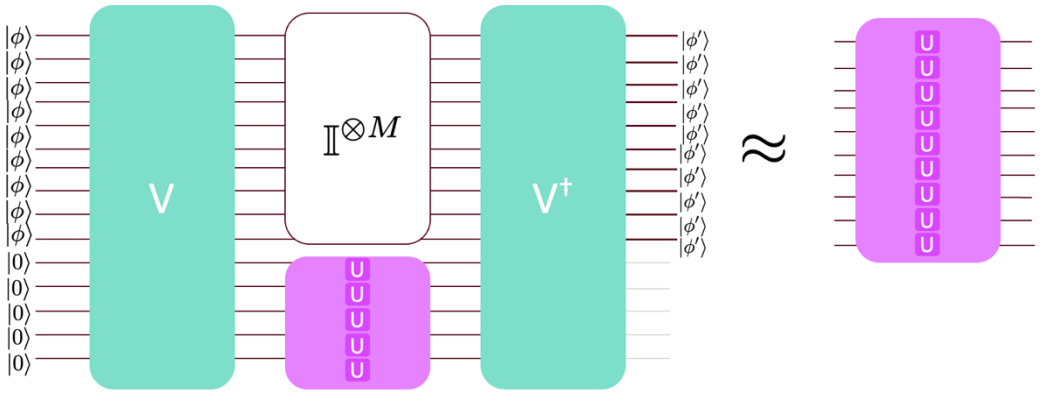
1. **Semidefinite programming** (numerics)
2. **Estimate and re-prepare**
3. **Dummy strategy** (use a fixed channel on (M-N) systems)
4. **Coherent protocols**[13,14]



13. W. Dür, P. Sekatski, and M. Skotiniotis. "Deterministic Superreplication of One-Parameter Unitary Transformations". *Phys. Rev. Lett.* 114, 120503 (2015). doi:10.1103/PhysRevLett.114.120503
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Estimate and prepare

"Brute force" approach

> *"Independently of all the complicated bounds derived so far, what happens if we just measure and reprepare a channel based on its estimator?"*

Estimate the channel using a process that outputs a classical output with a corresponding probability distribution, **then prepare M copies** of the estimated channel.

We show that: if you can estimate well, then you can replicate well

(super-linear mean squared error implies super-linear replication)

If you have a protocol that has a super-linear mean squared error for channel estimation, then you have a process with with a super-linear replication rate. (Concavity of fidelity > Jensen's inequality > mean squared error).

Super-replication achieved

For U(1), SU(2) and SU(d), the **estimate and prepare** strategy achieves super-replication---we know what measurements and what states lead to optimal estimation. For bit flips before/after phase gates, we have protocols.

Super-replication of all qubit phase gates

We have a protocol for

$$\mathcal{P}_2 := \{\mathcal{U}_\theta \,|\, \theta \in [0, 2\pi)\}$$
$$\mathcal{U}_\theta[\,\cdot\,] = U_\theta[\,\cdot\,]U_\theta^\dagger := e^{i\theta\sigma_z} \cdot e^{-i\theta\sigma_z}.$$

Super-replication of all qubit unitaries

We have a protocol for

$$\mathcal{U}_2 = \{\mathcal{U}[\,\cdot\,] := U \cdot U^\dagger | U \in \mathrm{SU}(2)\}$$

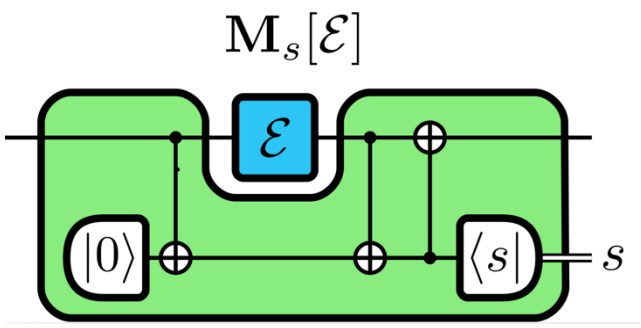
Super-replication of bit flip noise before or after the phase gate

Noisy phase gates can be super-replicated

$$\mathcal{A} := \{\mathcal{A}_\theta = \mathcal{X}_p \circ \mathcal{U}_\theta | \theta \in [0, 2\pi)\}$$
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$$\mathcal{U}_\theta[\cdot] = U_\theta[\cdot]U_\theta^\dagger := e^{i\theta\sigma_z} \cdot e^{-i\theta\sigma_z}.$$
$$\mathcal{X}_p[\cdot] = (1-p)\cdot + p\,\sigma_x \cdot \sigma_x$$

Summary and outlook

Cloning quantum states is the same as cloning the trash-and-replace channel.

- All cloning tasks can be seen as instance of channel cloning.

Lower bound for the distance between two channels via estimation. Consequences:

- Sufficient conditions for no super-replication of a family of channels.
- Super-replication of $SU(d)$ has zero robustness.
- No super-replication of classical channels, the full set of noisy quantum gates or the amplitude damping channel.

Future research

- Rephrase replication/cloning at the level of continuous semi-group dynamics. Does there exist a process that, given access to a system evolving under a given parametrised family of Liouvillian dynamics for a total time t, can simulate its evolution for a time $t' > t$.
- Can we efficiently solve some of the SDPs? (symmetries)
- **Can we do cryptography with this?**
- **Can we prove similar results with thermodynamics?**

Thank you for your attention

Channel cloning question.

Consider an individual interacting with ChatGPT.

One interaction with ChatGPT (asking a question and receiving an answer) **= one query of an unknown channel.**

- The individual would like to ask M independent questions
- Their subscription plan only allows for N<M queries.
- A general way to do so could be offered by a channel cloning process, which could embed the N queries into a larger process into which one can input M queries and receive M corresponding answers.

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Can ChatGPT be super-replicated? I.e., can $M \sim N^2$?

No. ChatGPT is a stochastic channel, and those cannot be super-replicated. We do not know the best rate though, it could still be super-linear.

Appendix

Figure of merit

Channel closeness

Recall for states

Figure of merit for the closeness of quantum states was the fidelity $F(\rho,\sigma) := \mathrm{tr}\,|\sqrt{\rho}\sqrt{\sigma}| = \mathrm{tr}\,\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}}$

Optimal cloning fidelity:

$$\mathsf{F}^{\mathcal{S}}(N,M) := \max_{\mathcal{P}} \inf_{\rho\in\mathcal{S}} F\left(\mathcal{P}[\rho^{\otimes N}], \rho^{\otimes M}\right)$$

For channels

CJ fidelity

$$F_{\mathrm{CJ}}(\mathcal{A},\mathcal{B}) := F((\mathrm{id}\otimes\mathcal{A})[\Phi^+], (\mathrm{id}\otimes\mathcal{B})[\Phi^+])$$

where $\Phi^+ := \frac{1}{d}\sum_{ij}|ii\rangle\langle jj|$ Is the max ent. state

Diamond fidelity

$$F_\diamond(\mathcal{A},\mathcal{B}) := \min_{\Psi} F\big((\mathrm{id}\otimes\mathcal{A})[\Psi], (\mathrm{id}\otimes\mathcal{B})[\Psi]]\big)$$

where min is over all pure states.

Diamond norm

$$\tfrac{1}{2}||\mathcal{A}-\mathcal{B}||_\diamond = \sup_{\Psi} ||(\mathrm{id}\otimes\mathcal{A})[\Psi] - (\mathrm{id}\otimes\mathcal{B})[\Psi]||$$

related to **diamond fidelity distance** by F-v d G.

Fidelity is not a proper distance. Easy fix: take inverse cosine $\bullet \in \{\mathrm{CJ}, \diamond\}$

$$D_\bullet := \arccos F_\bullet \in \left[0, \tfrac{\pi}{2}\right].$$

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$$1-\cos D_{\diamond}(\mathcal{A},\mathcal{B}) \le \tfrac{1}{2}||\mathcal{A}-\mathcal{B}||_{\diamond} \le \sin D_{\diamond}(\mathcal{A},\mathcal{B})$$

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Important conceptual interlude

❌ Cloning channels is not the same as cloning the CJ state of the channel!

> *"Why don't you just take the channel, do the CJ isomorphism on it and clone those states?"*

We are interested in N to M cloning: with N resources you can use each one only once. Take the route above and you are left with a pile of states that encode information about the channel, but the action of the channel is unrecoverable.

$$\mathrm{CJ}[\mathcal{E}] := (\mathrm{id} \otimes \mathcal{E})|\Phi^+\rangle\langle\Phi^+|$$

CJ state of the channel

N times



pile of states: channel action unrecoverable