On Entanglement in Quantum theory.

Yelena Guryanova
(Institute for Quantum Optics and Quantum Information, Vienna)
FH Technikum, Vienna

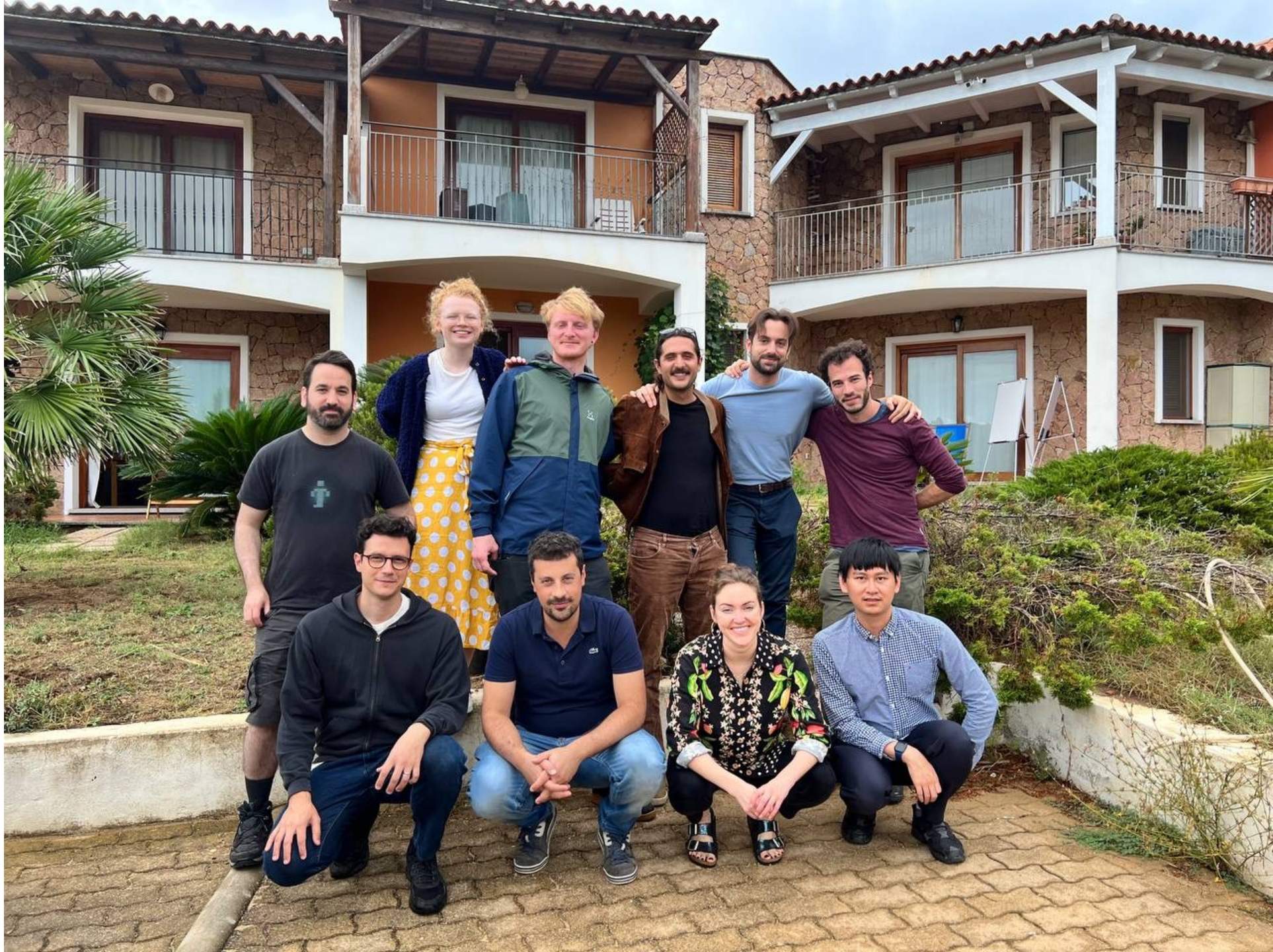
Monday 21tst August 2023.

universität wien

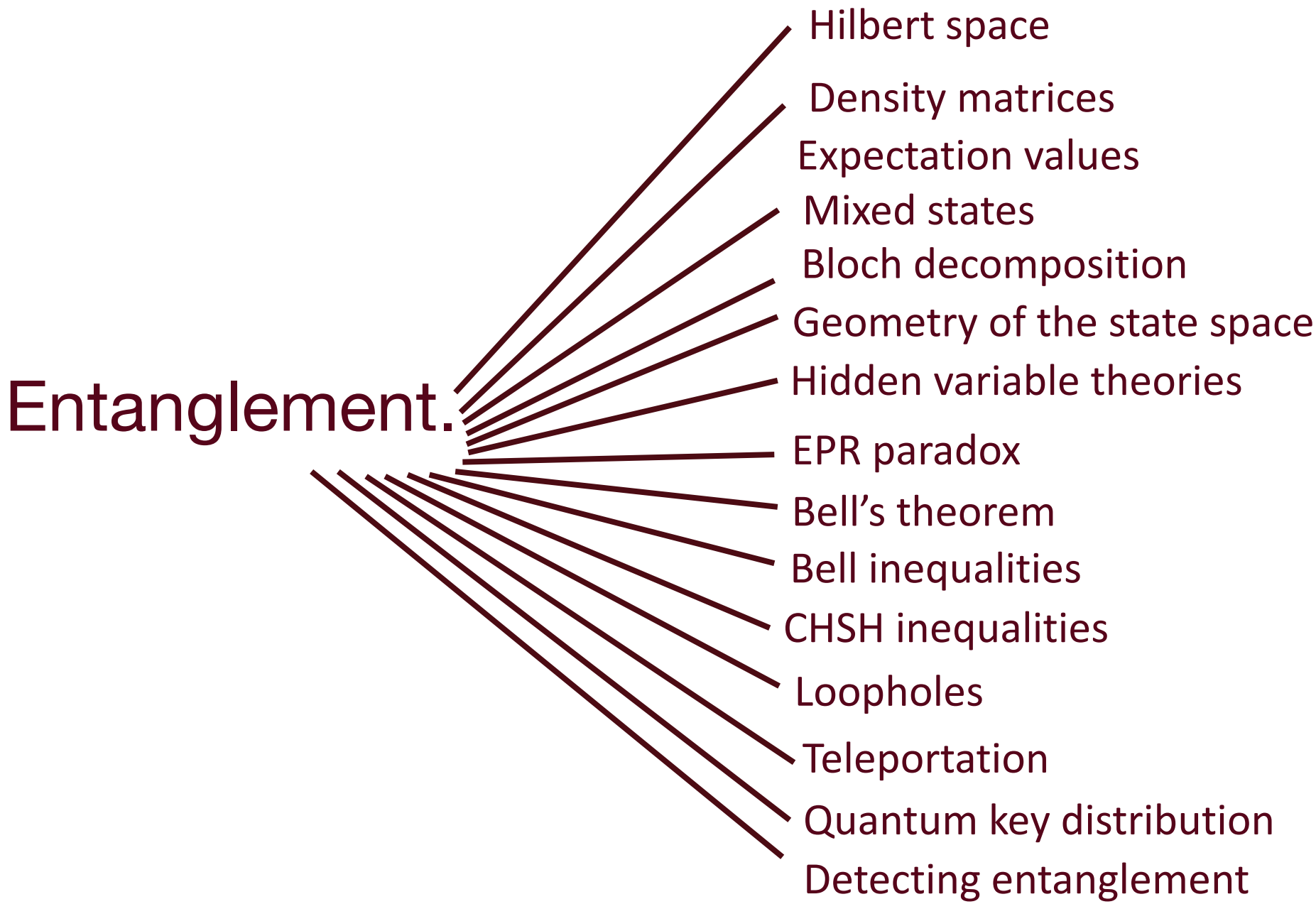
IQI

FWF

YIRG (Young independent researcher group) [Zukunftskollegs].



Entanglement.


Entanglement.
Hilbert space
Density matrices
Expectation values
Mixed states
Bloch decomposition
Geometry of the state space
Hidden variable theories
EPR paradox
Bell's theorem
Bell inequalities
CHSH inequalities
Loopholes
Teleportation
Quantum key distribution
Detecting entanglement

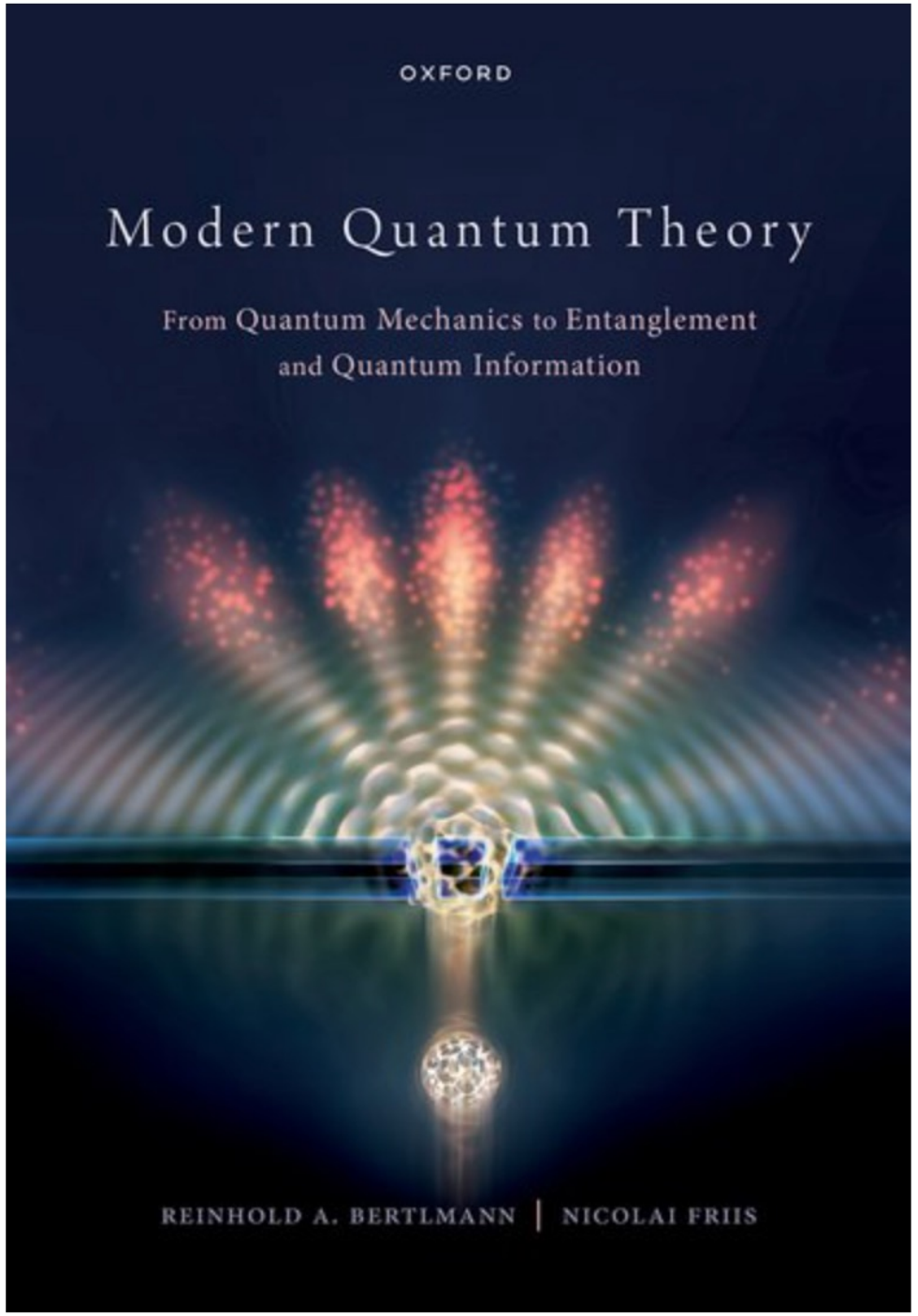
Modern Quantum Theory

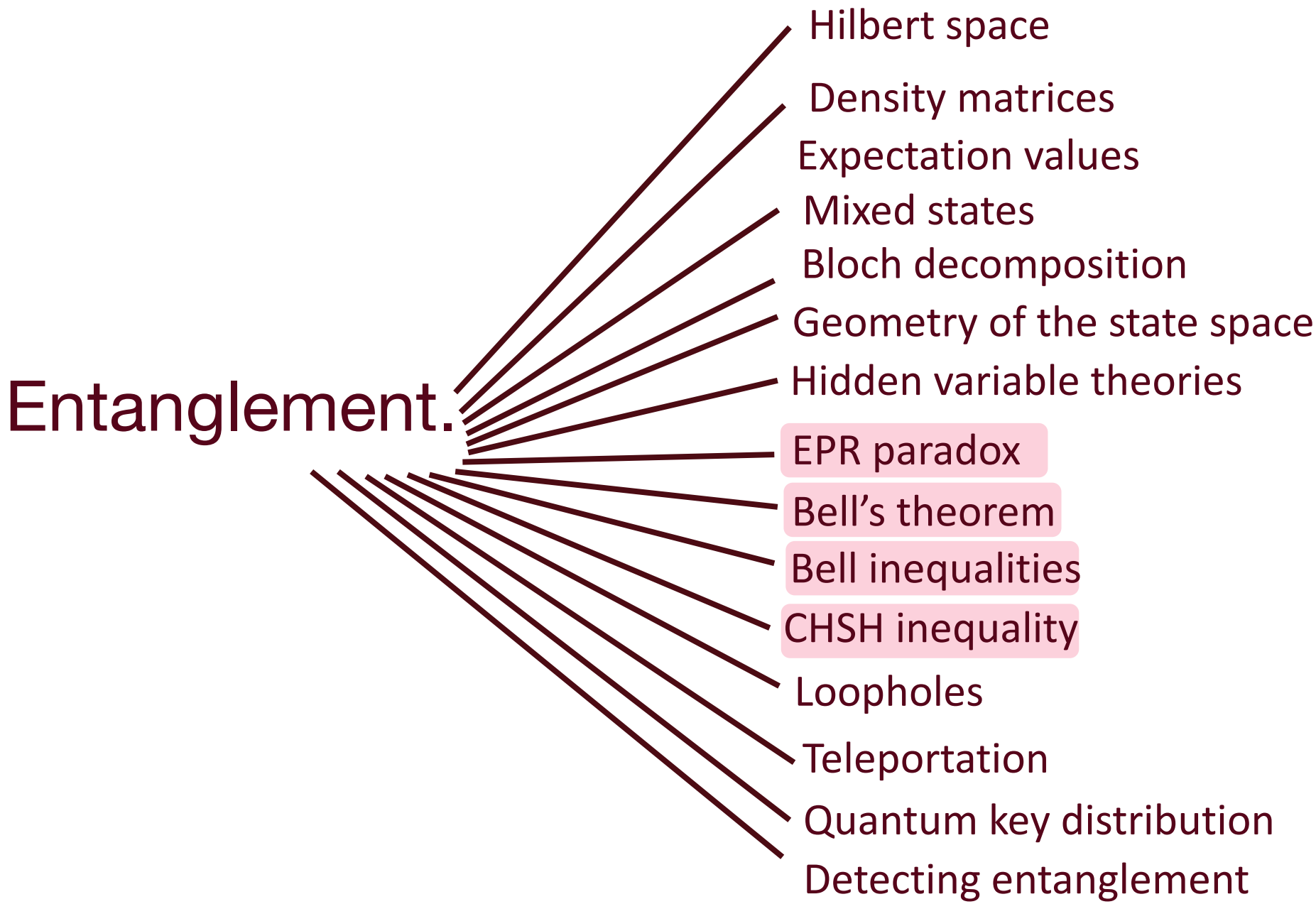
From Quantum Mechanics to Entanglement and Quantum Information

Prof Reinhold Bertlmann and **Dr Nicolai Friis**

- Combines introductory quantum mechanics with treatment of modern quantum information theory
- Based on a combination of theoretical experience and hands-on expertise
- Fully hyperlinked ebook including suggestions for further reading

Out 7th September 2023

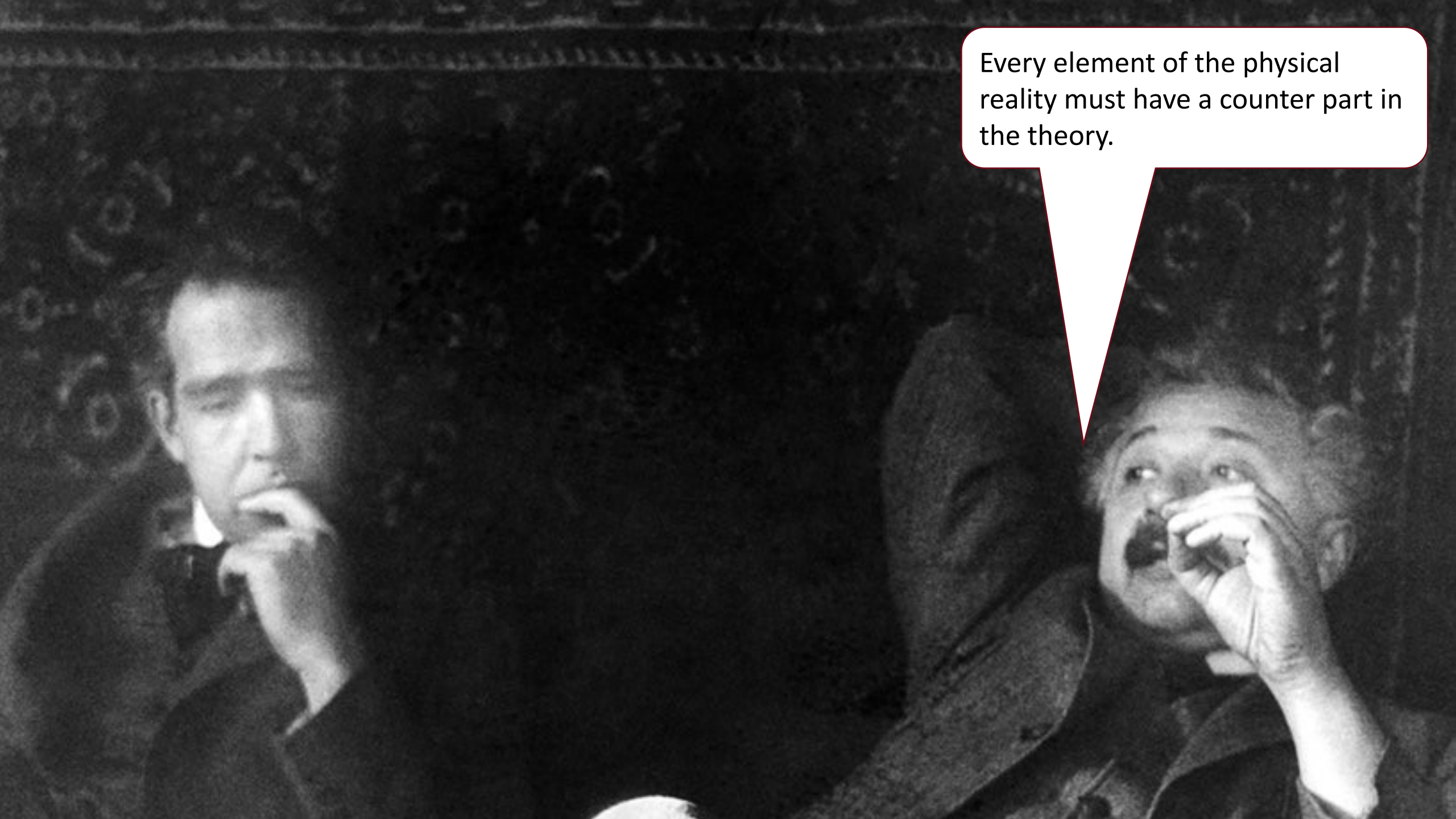


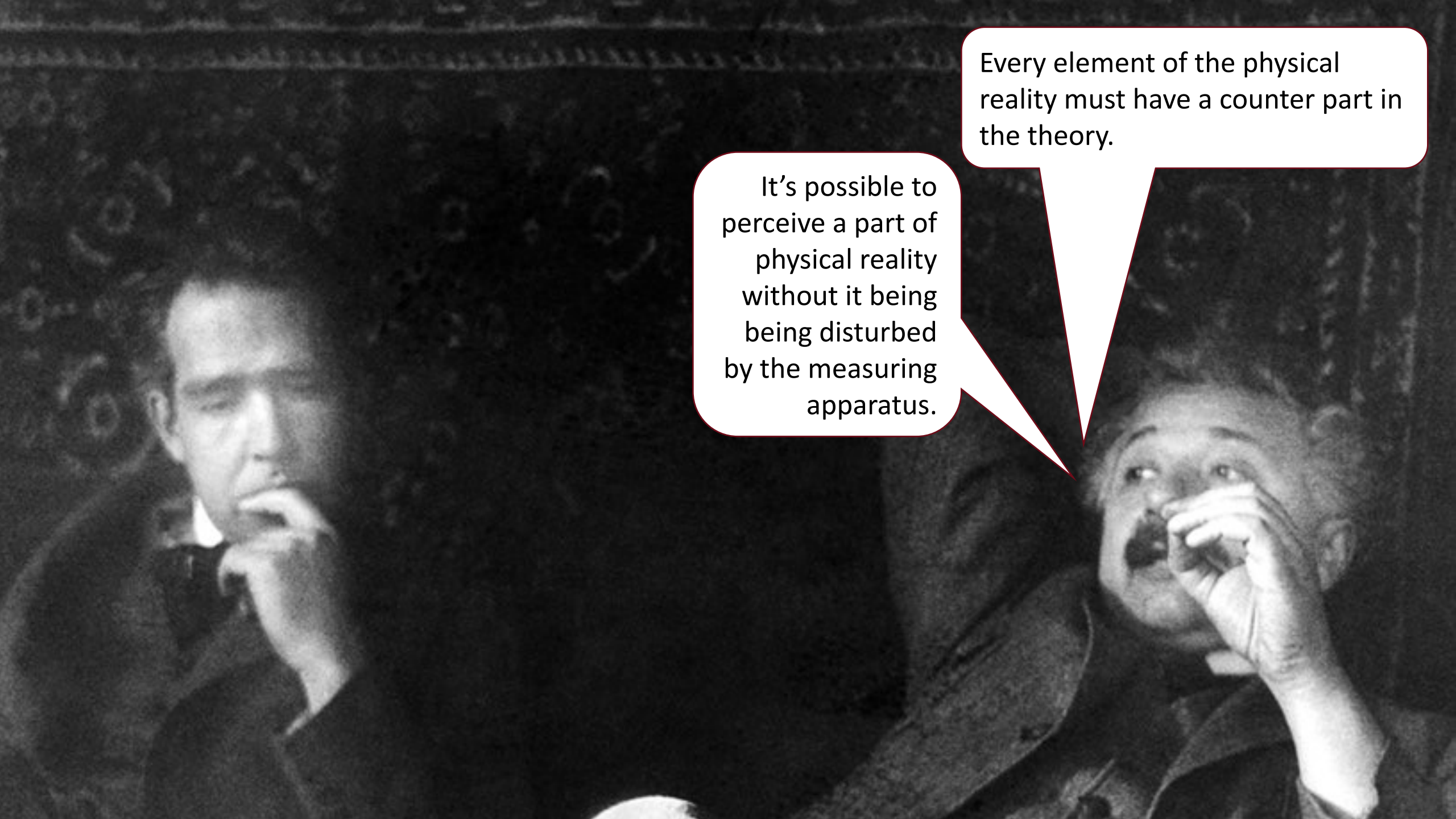

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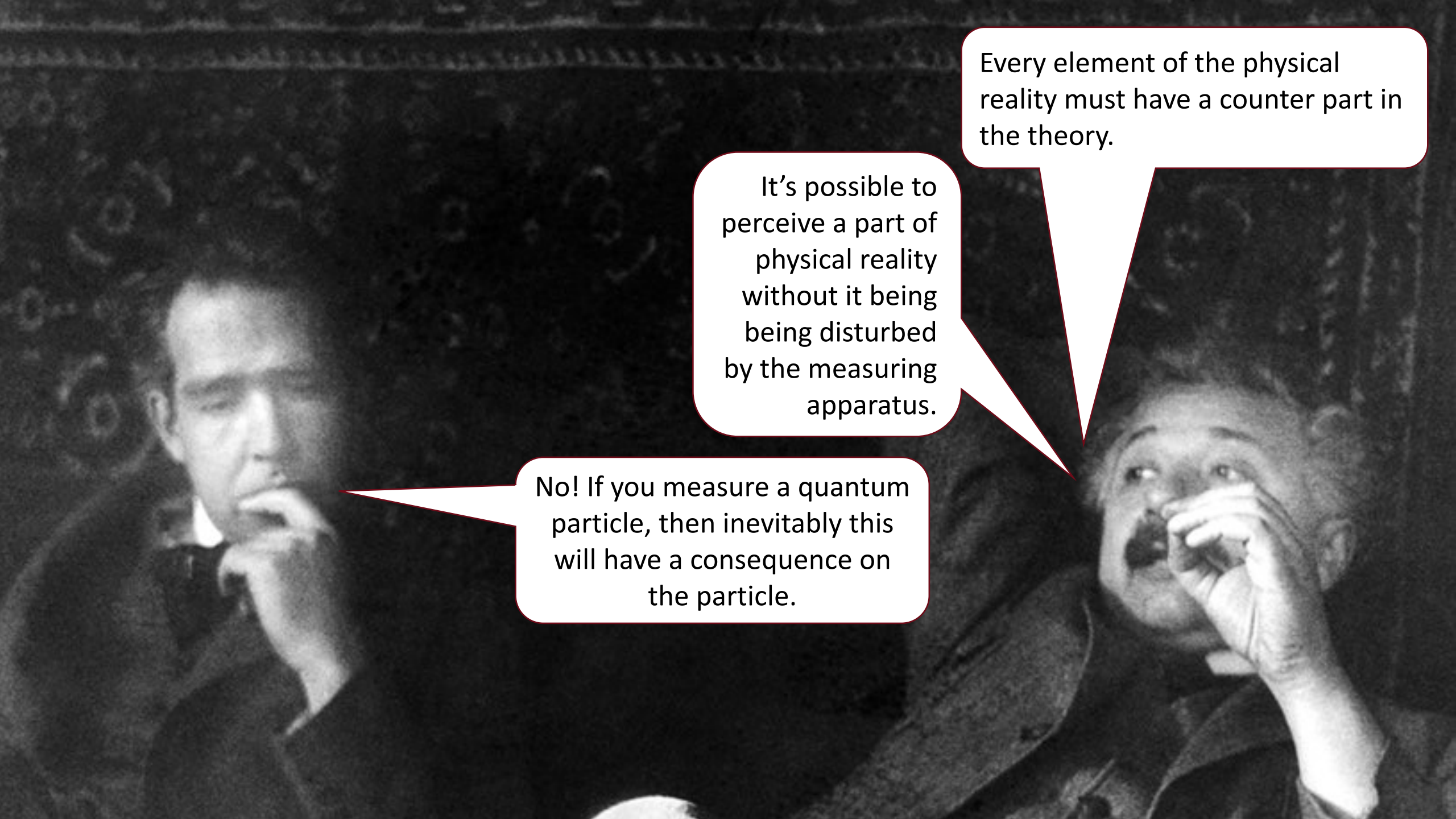
Quantum history condensed.

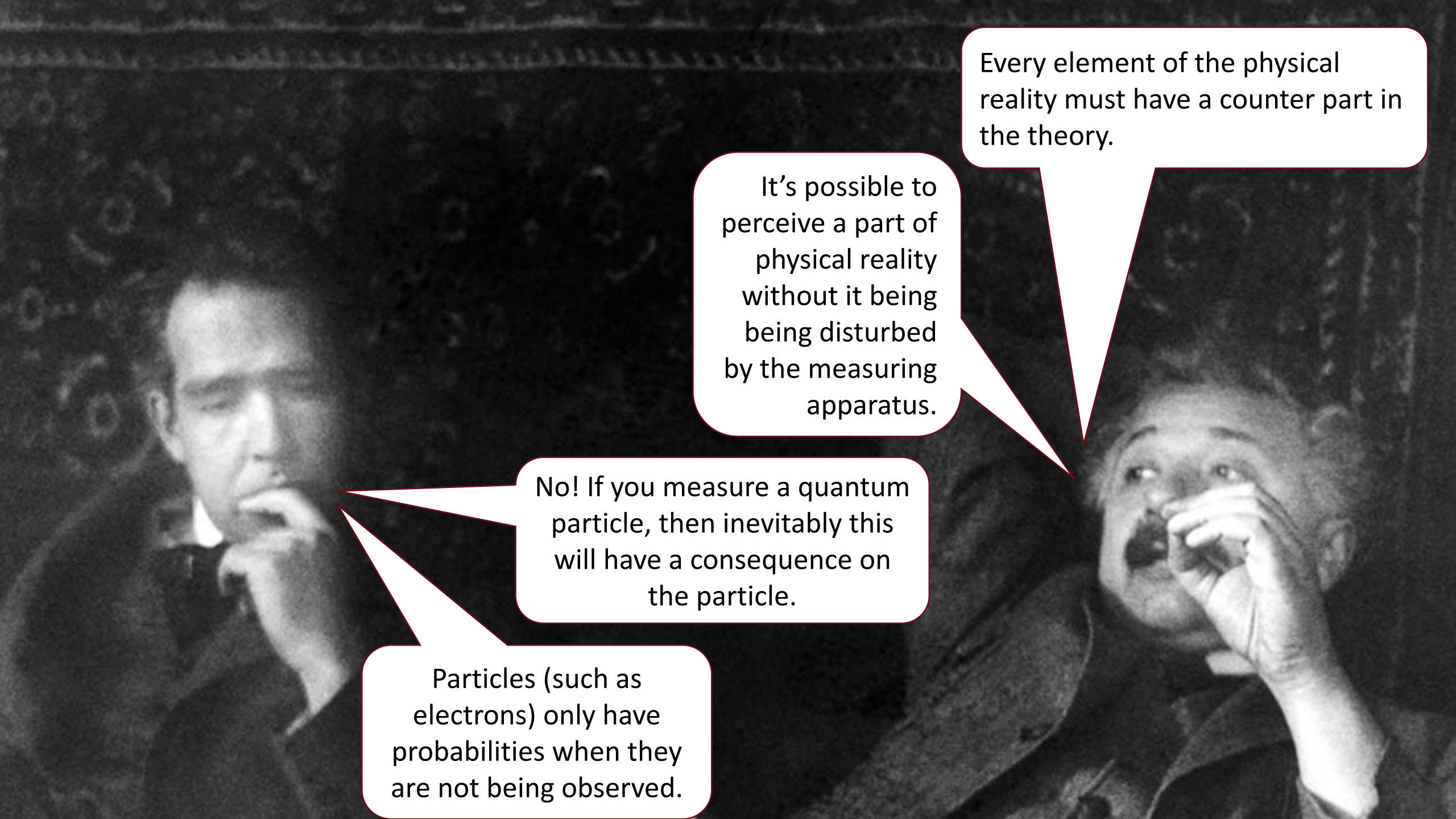


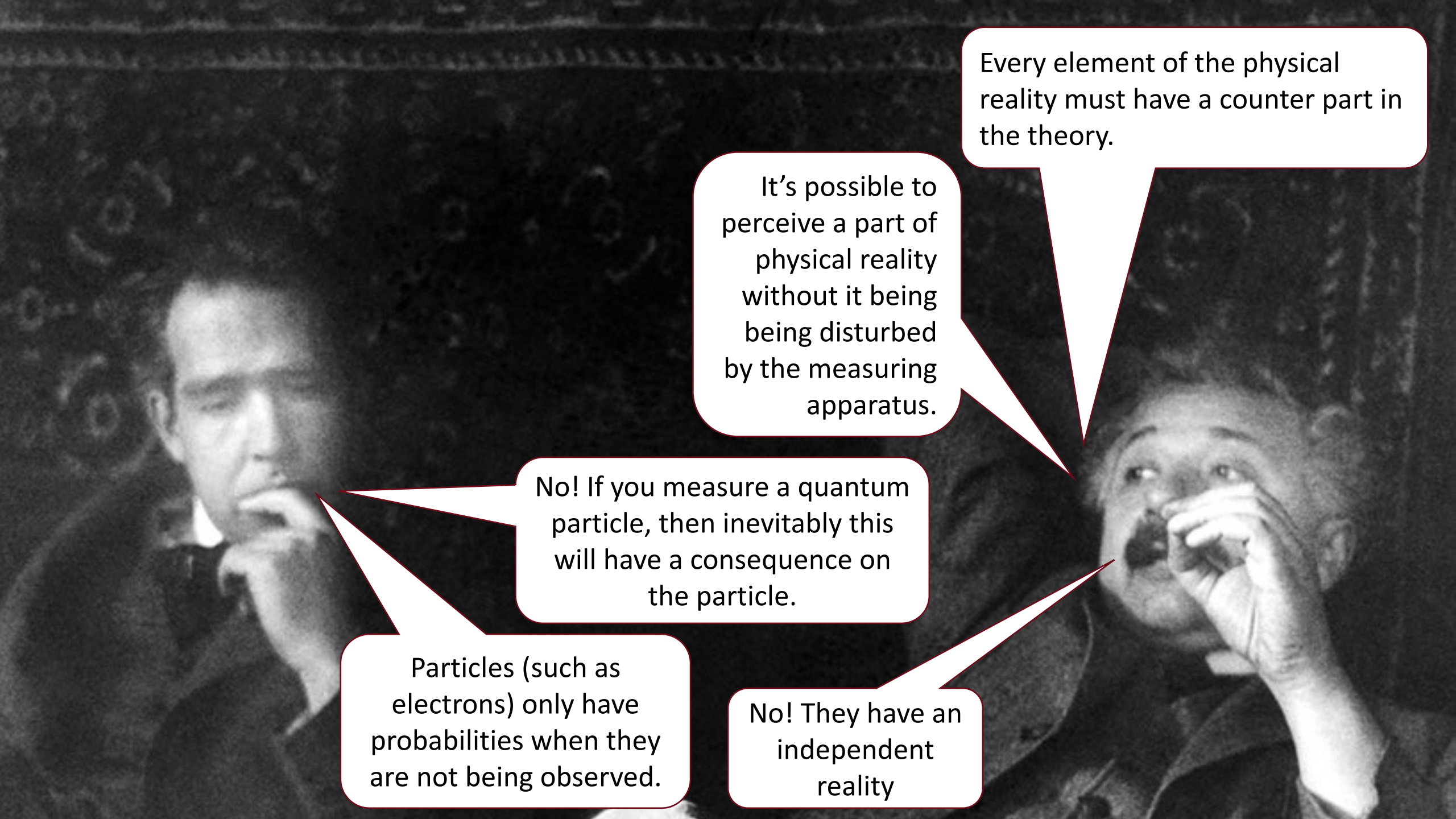



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No! If you measure a quantum particle, then inevitably this will have a consequence on the particle.
Particles (such as electrons) only have probabilities when they are not being observed.
No! They have an independent reality

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system



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Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

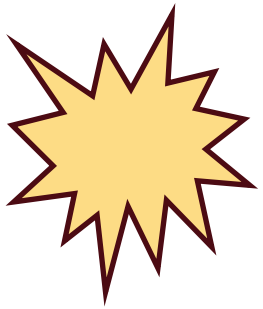
EPR thought experiment: assumptions

1. **Physical reality**
   "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to one) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

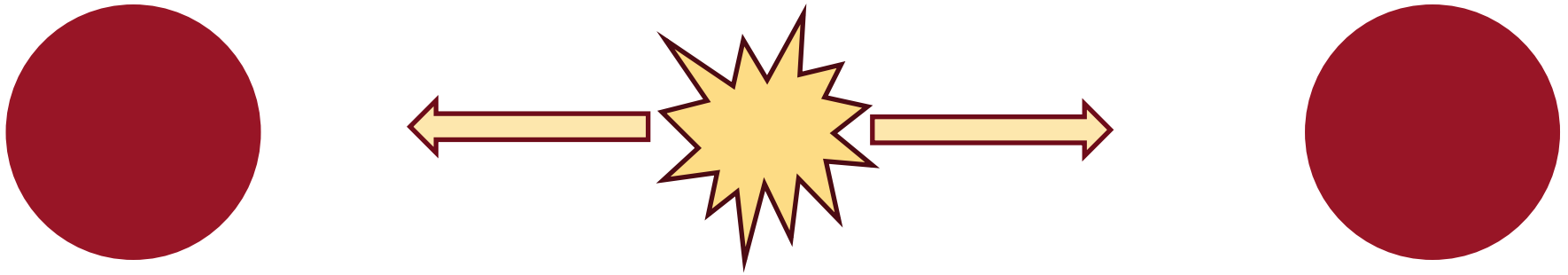
2. There is **no action at a distance** (we have local causality).

3. The **quantum-mechanical predictions are correct.**

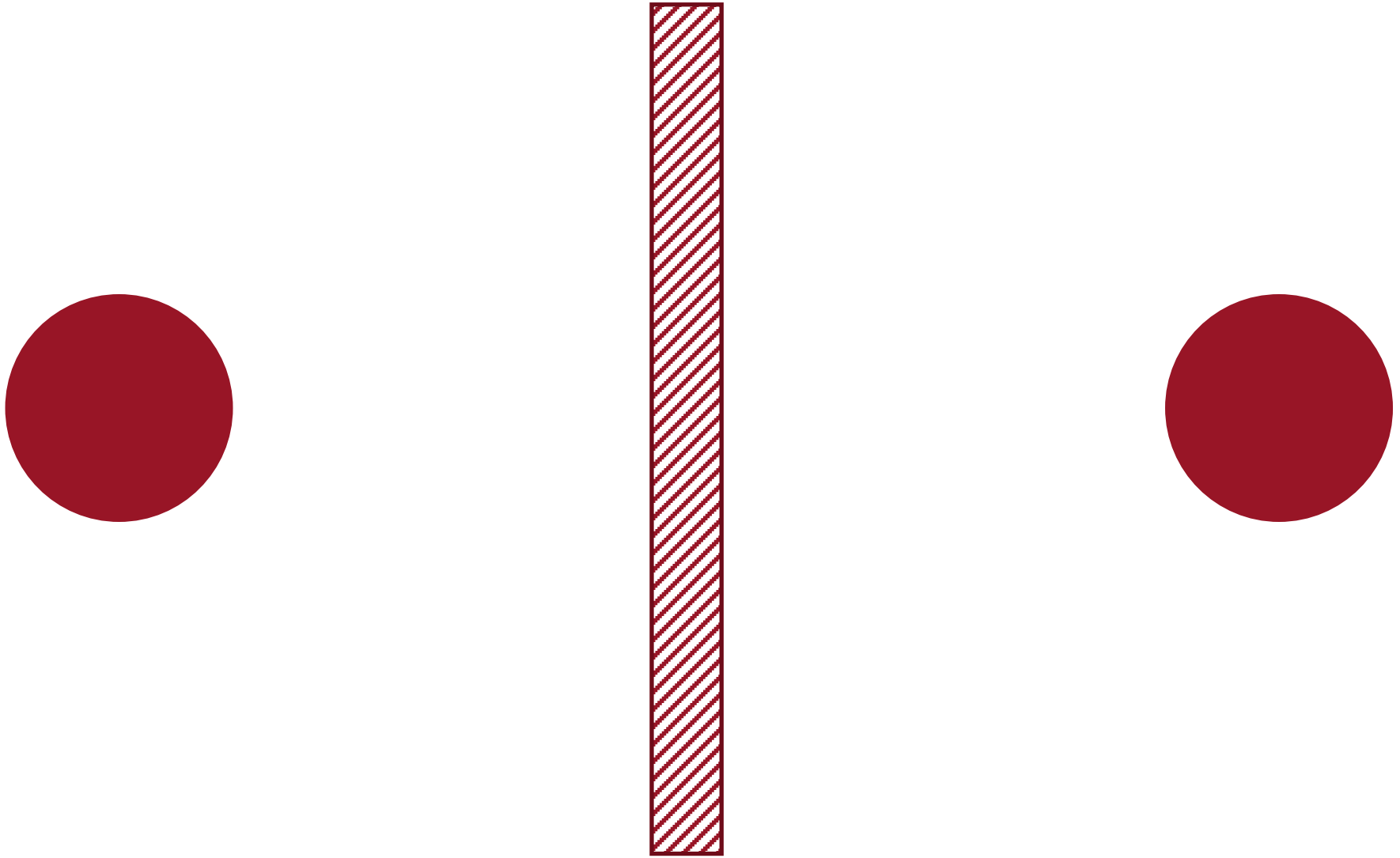
EPR thought experiment



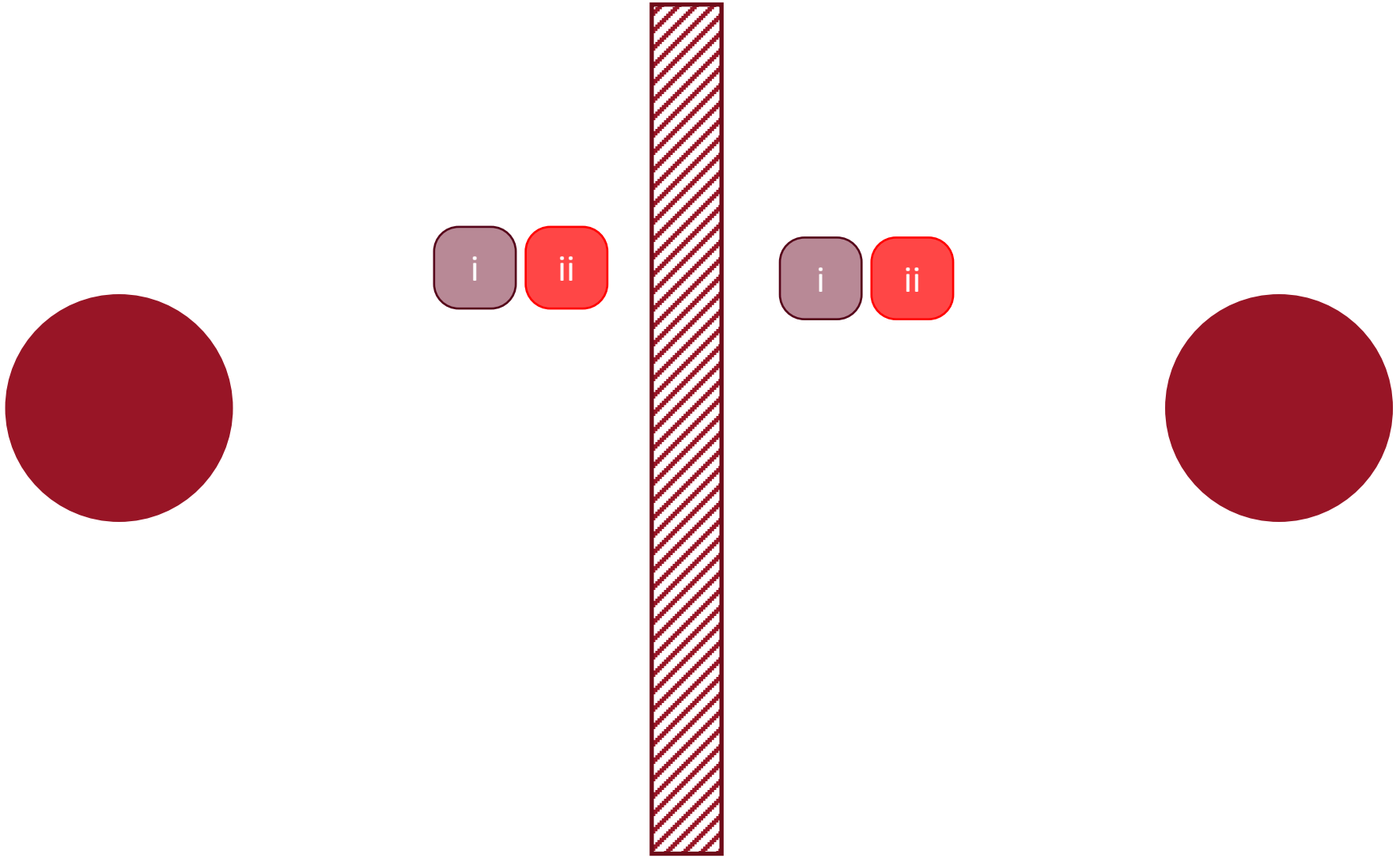
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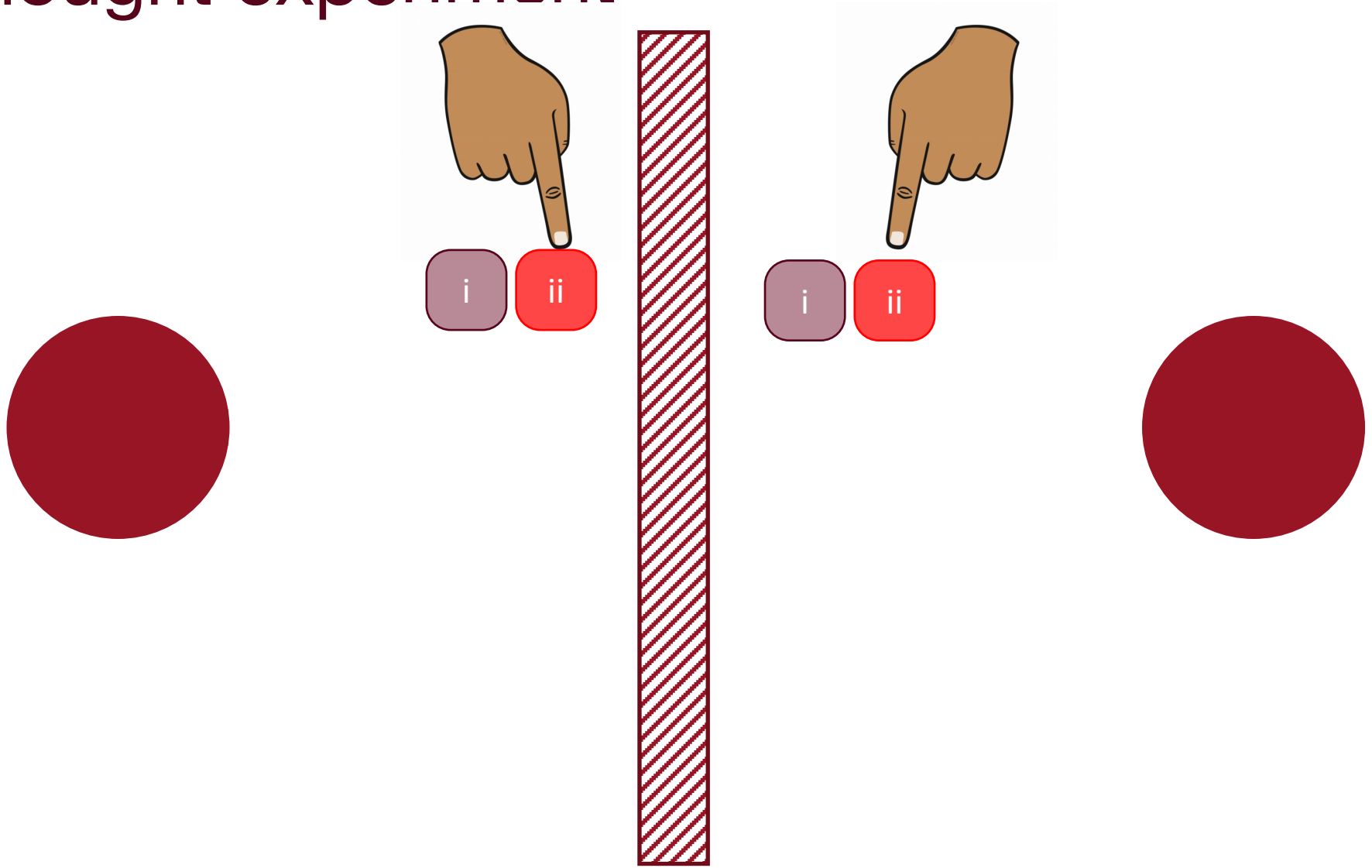
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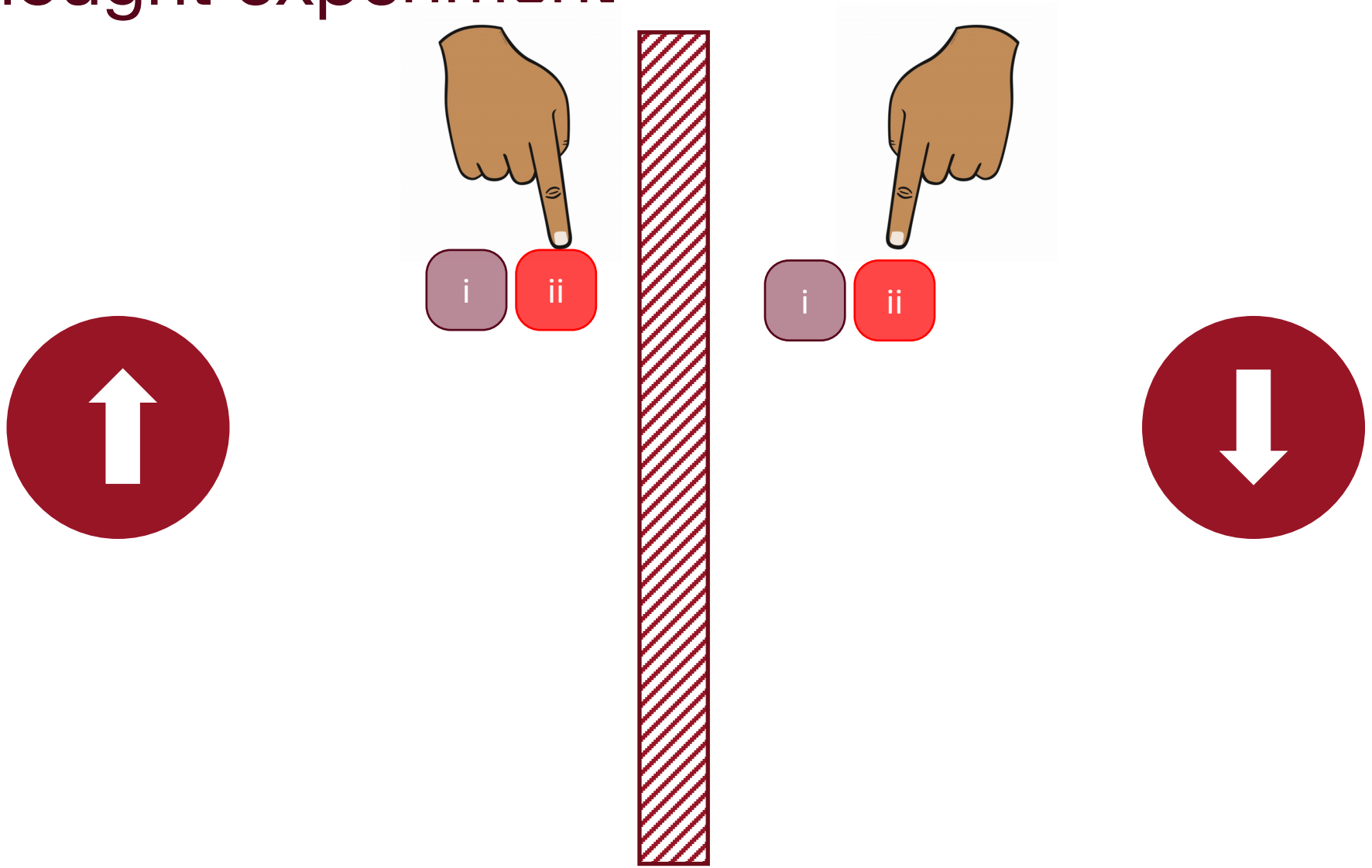
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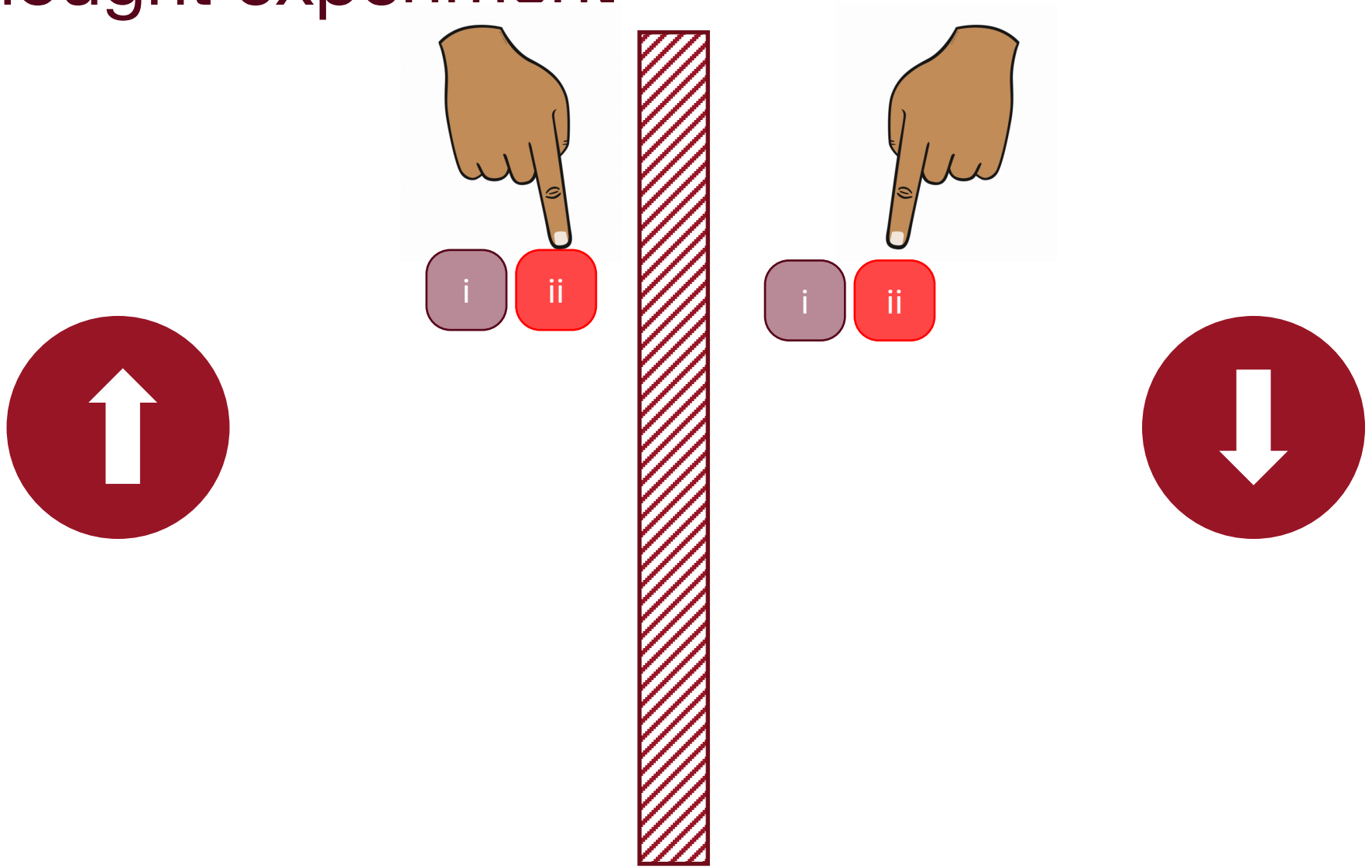
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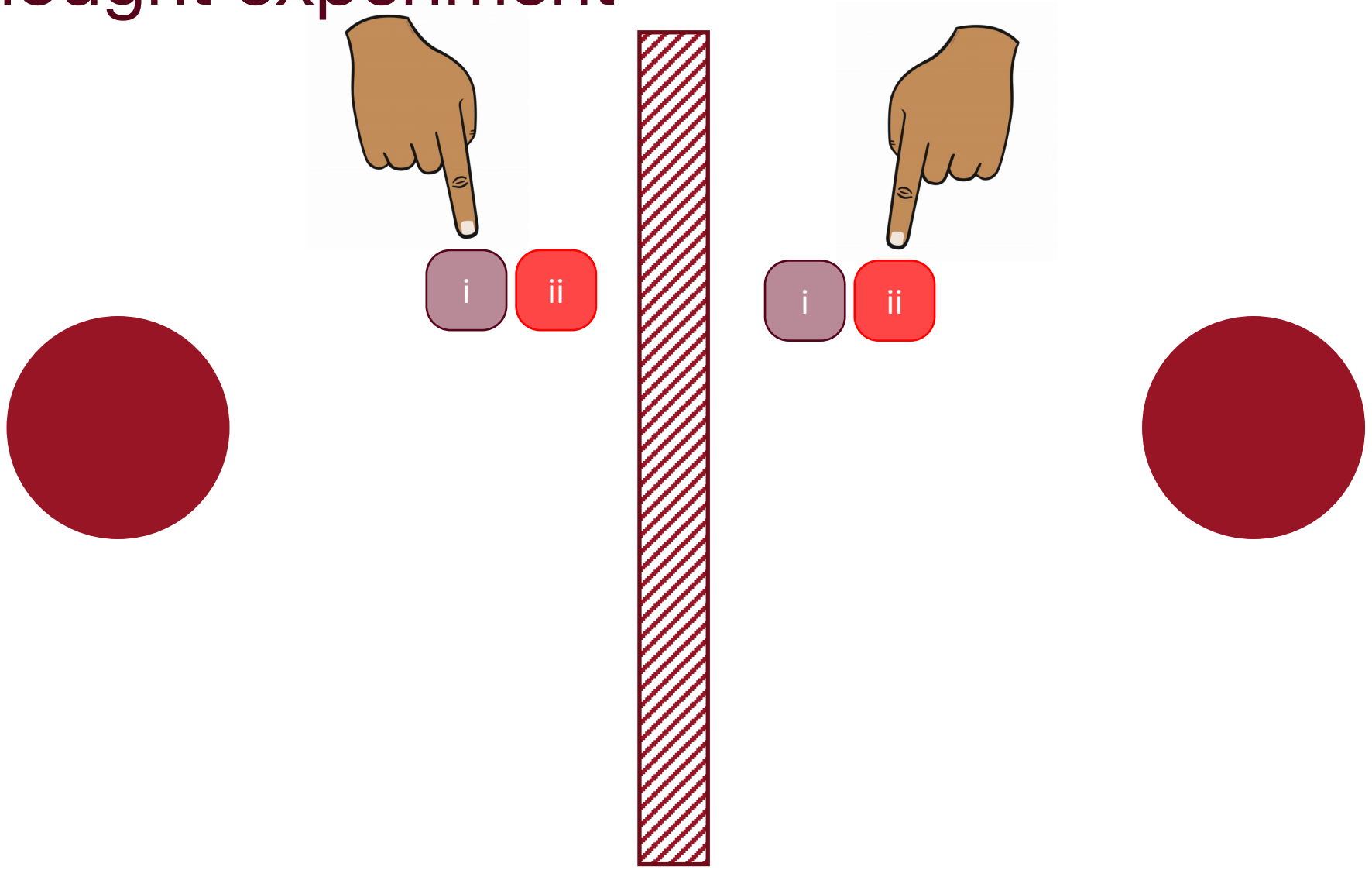
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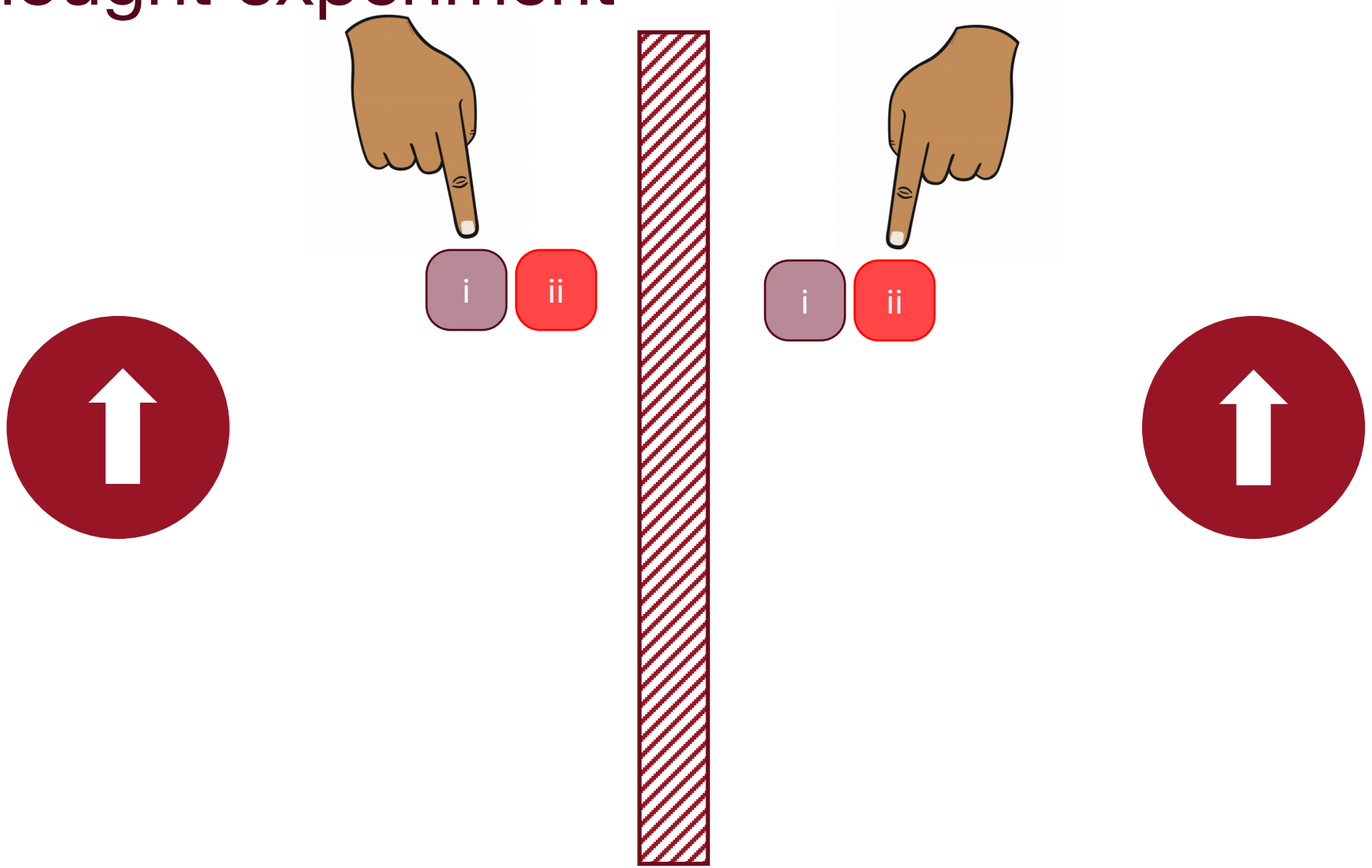
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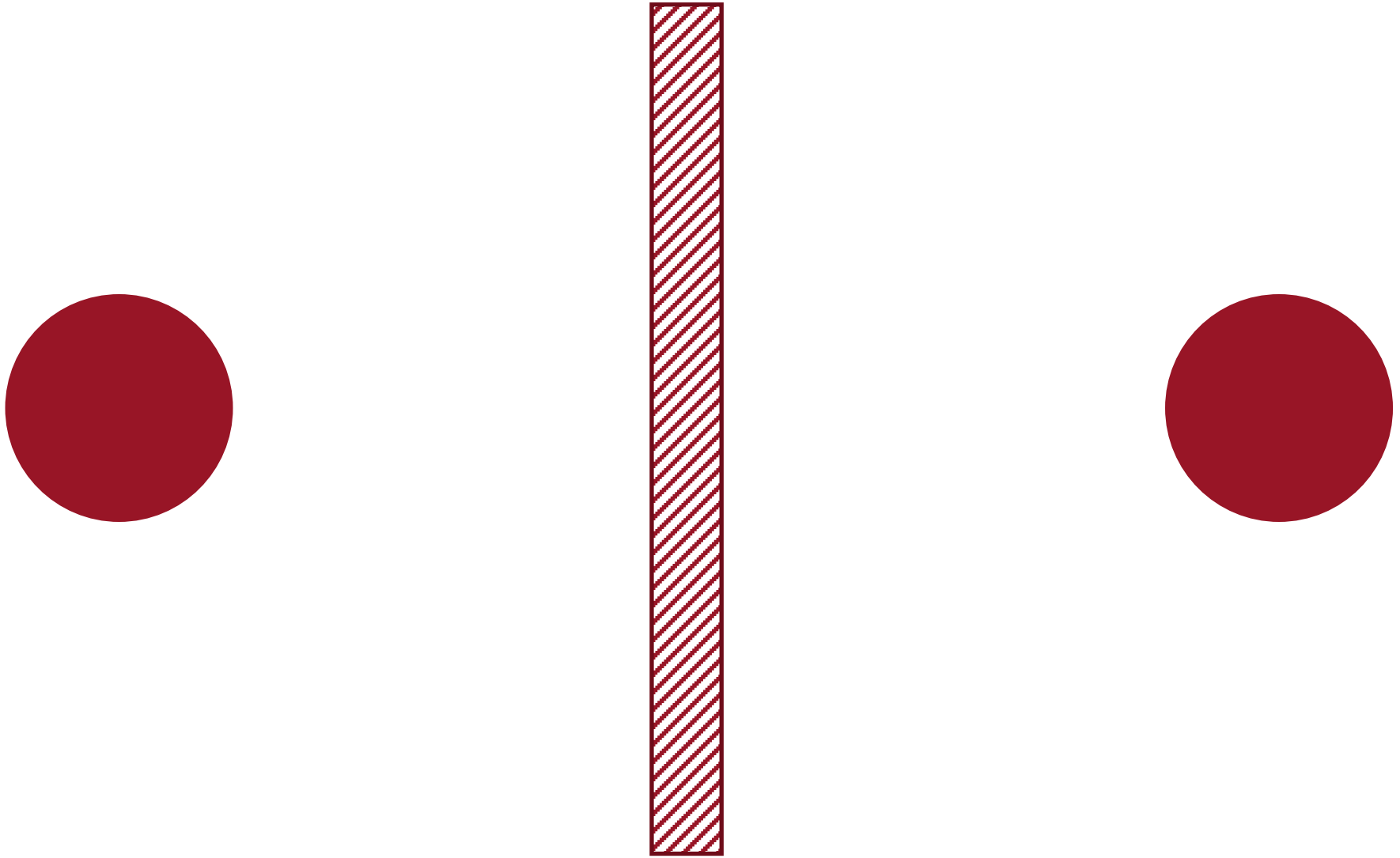
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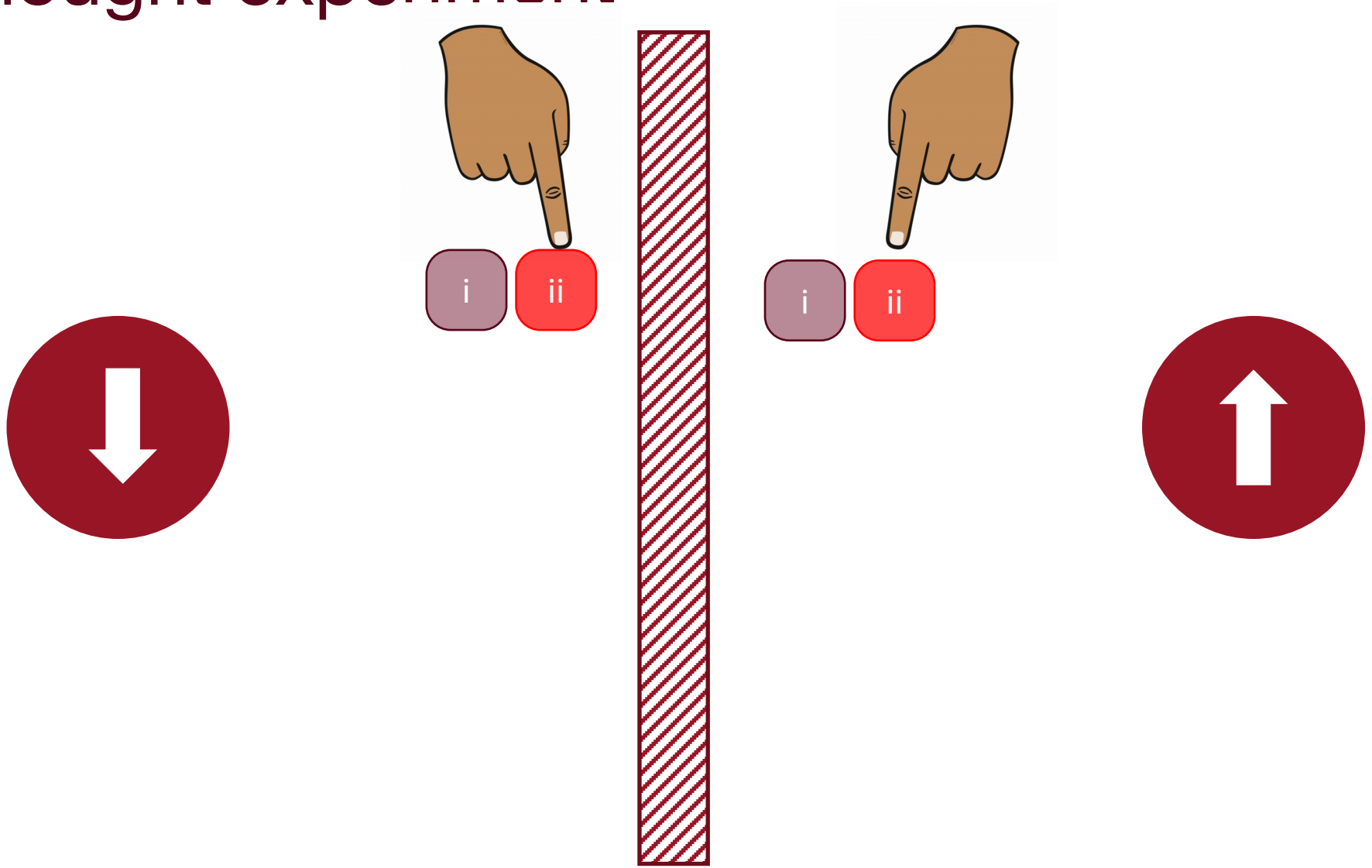
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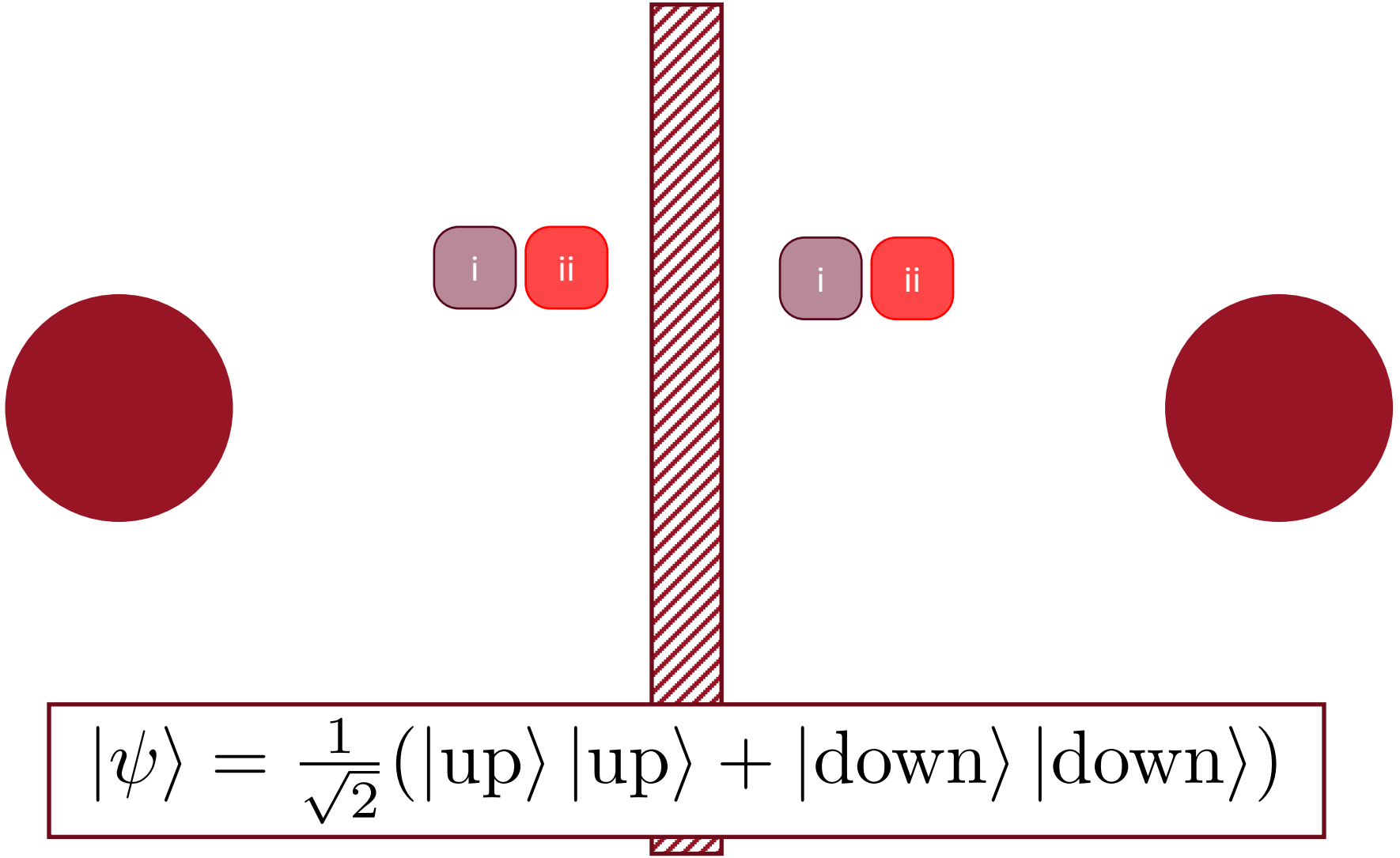
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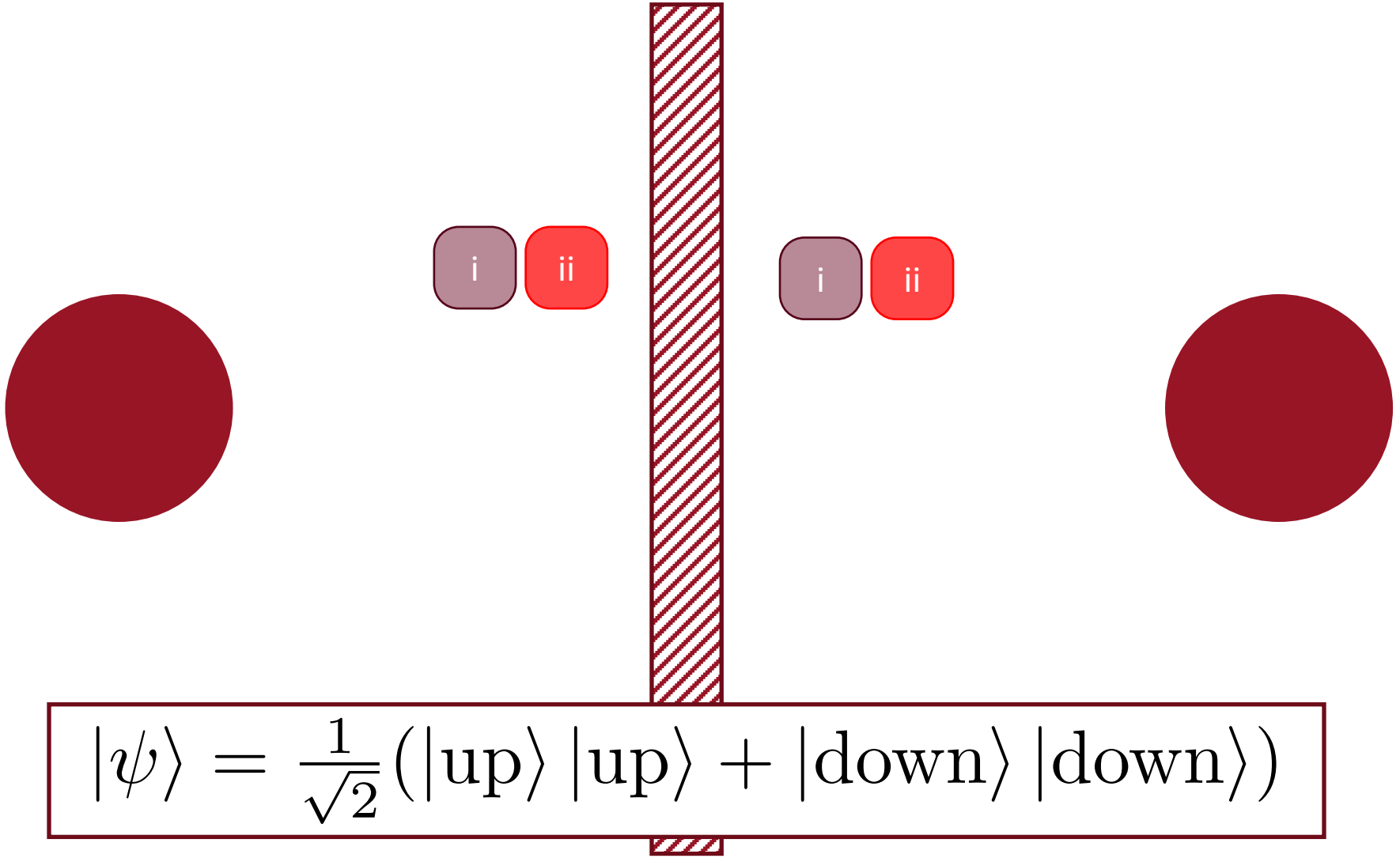


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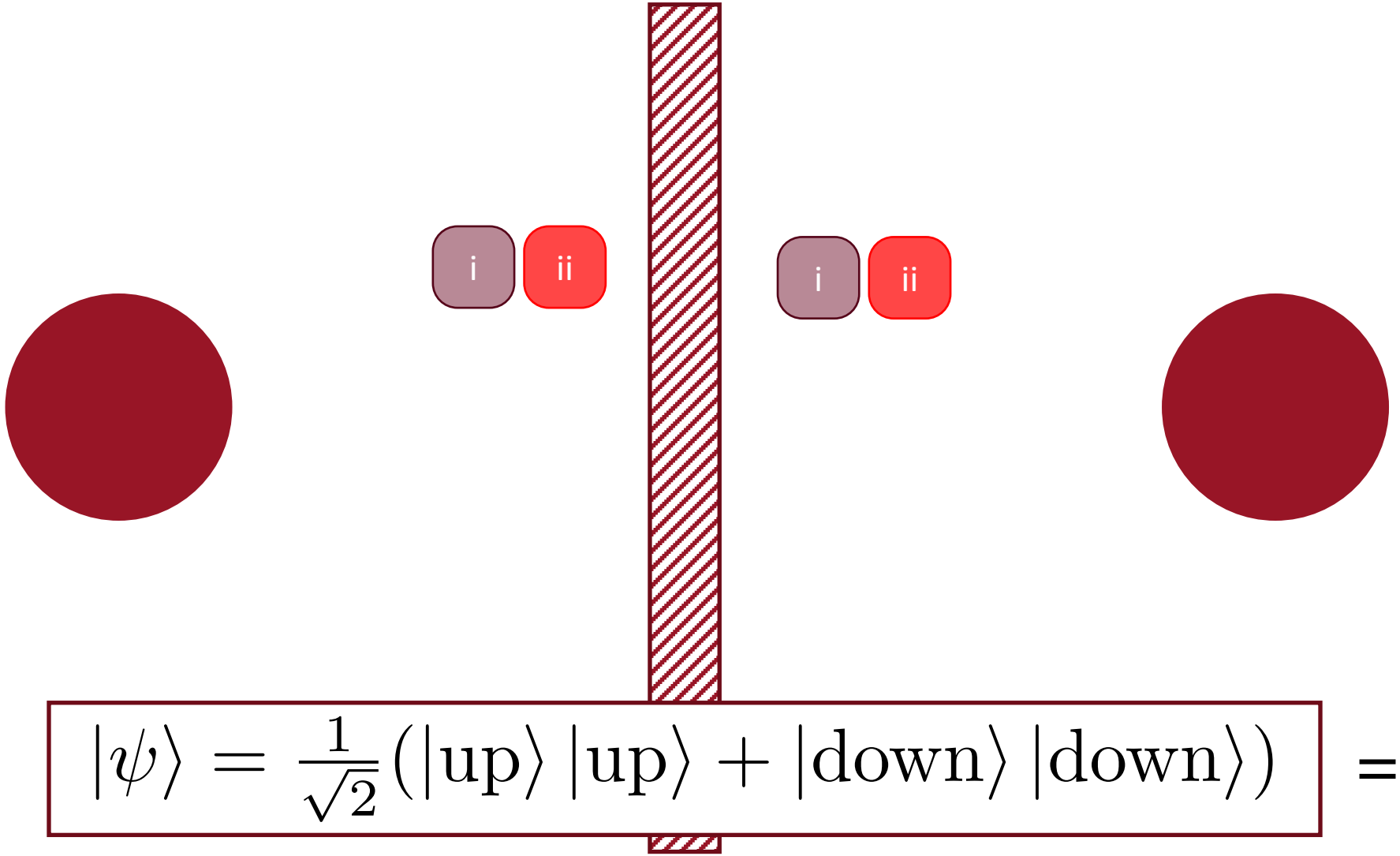
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle\,|\text{up}\rangle + |\text{down}\rangle\,|\text{down}\rangle)$$

EPR thought experiment



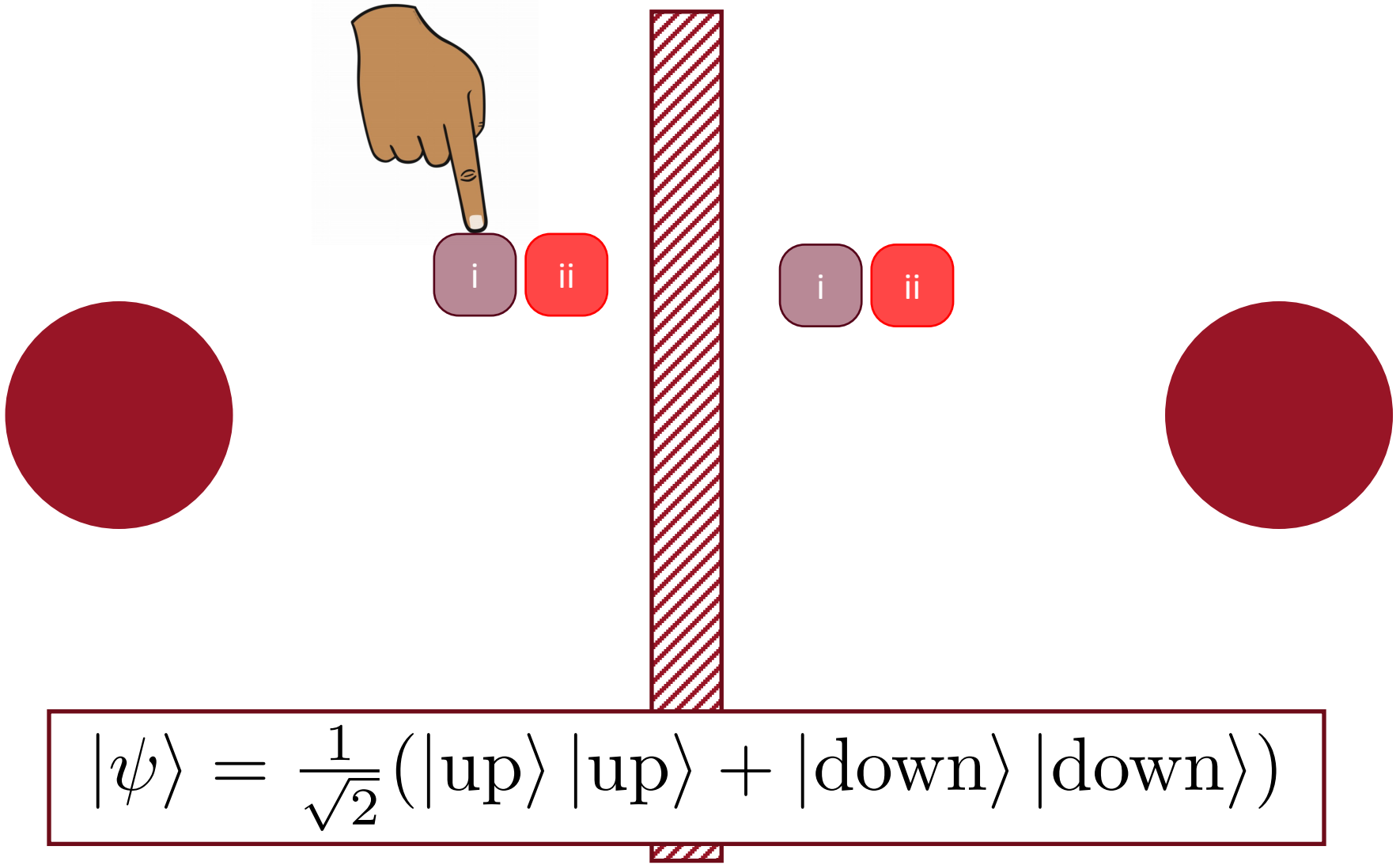
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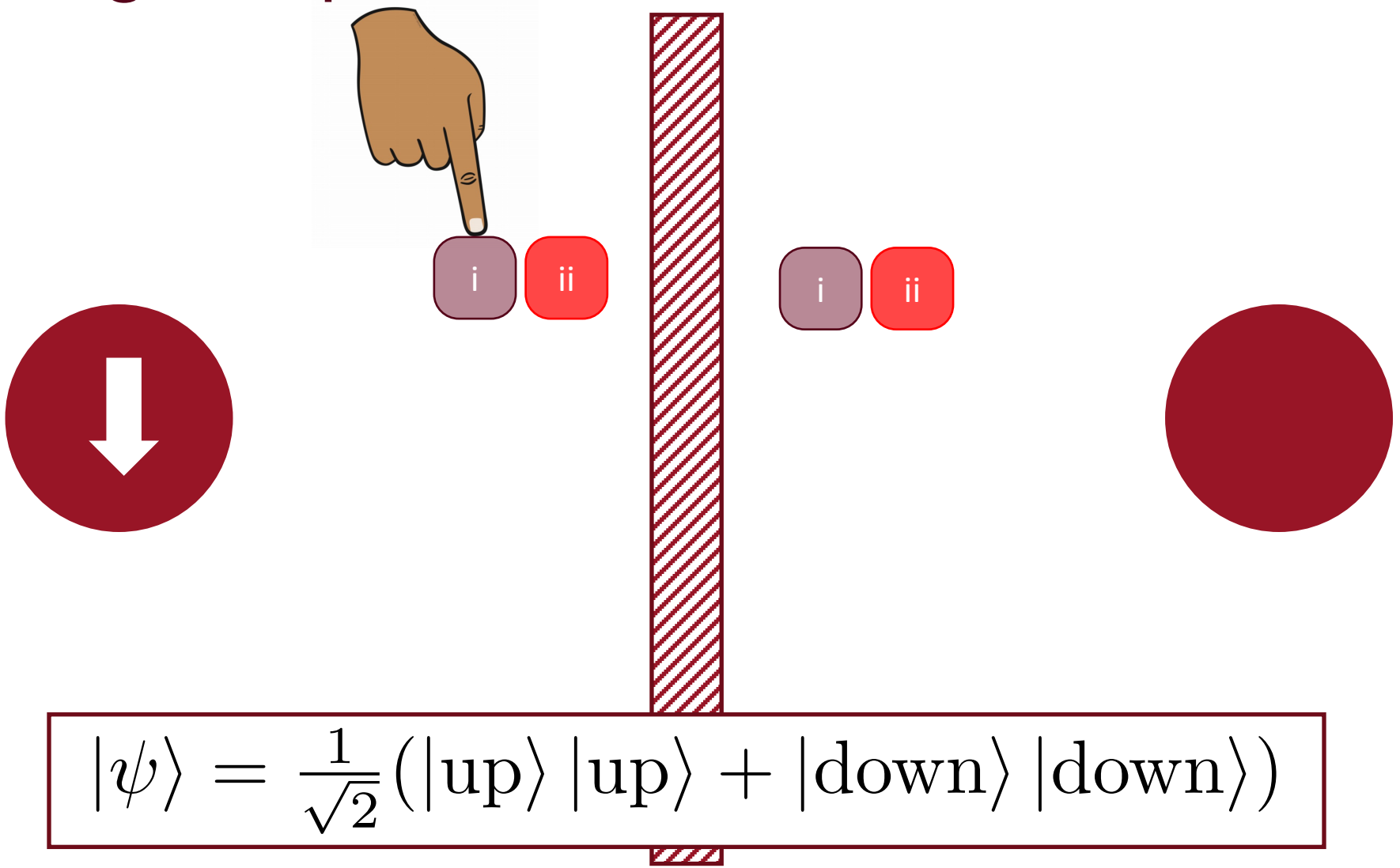
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle\,|\text{up}\rangle + |\text{down}\rangle\,|\text{down}\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

EPR thought experiment



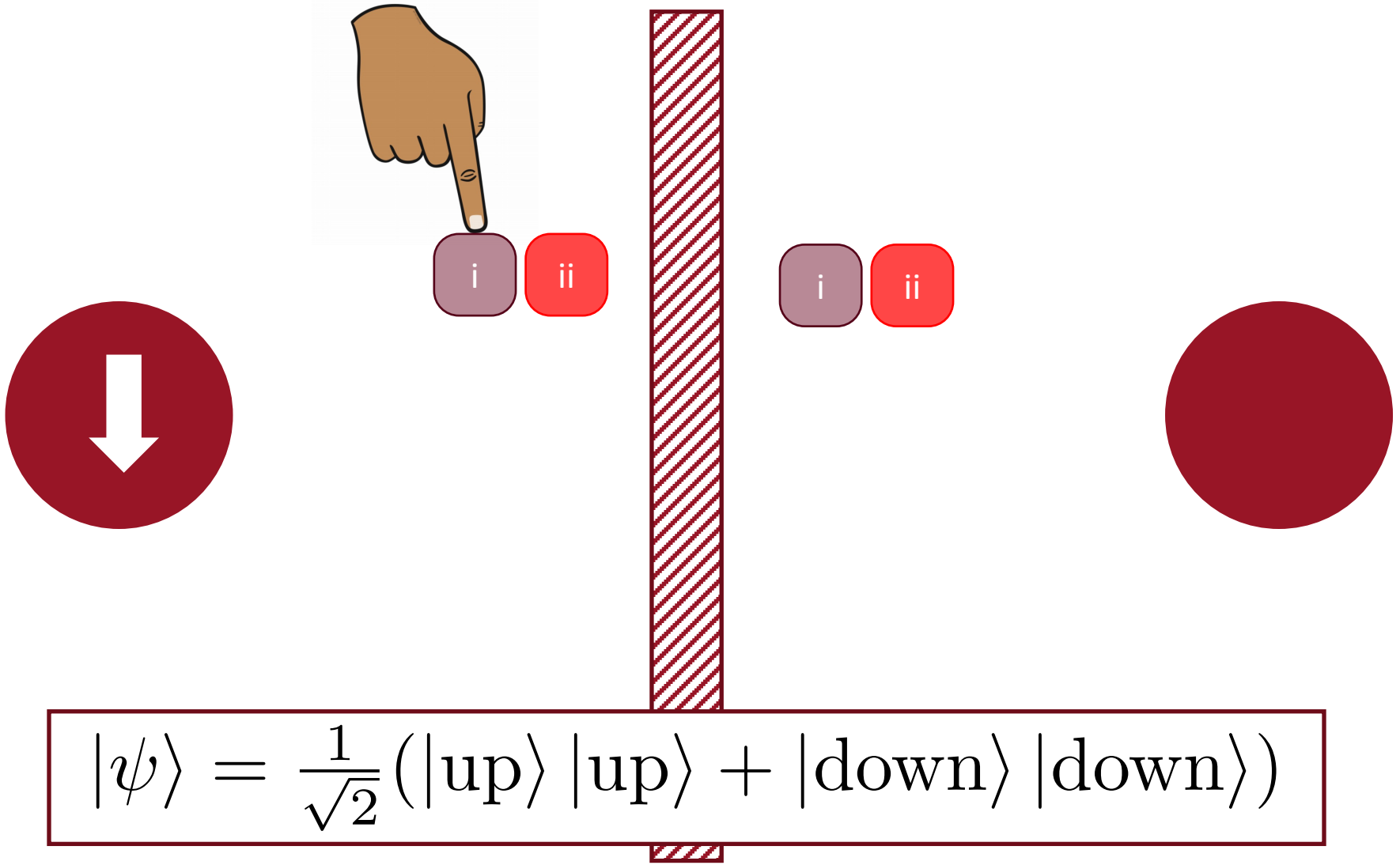
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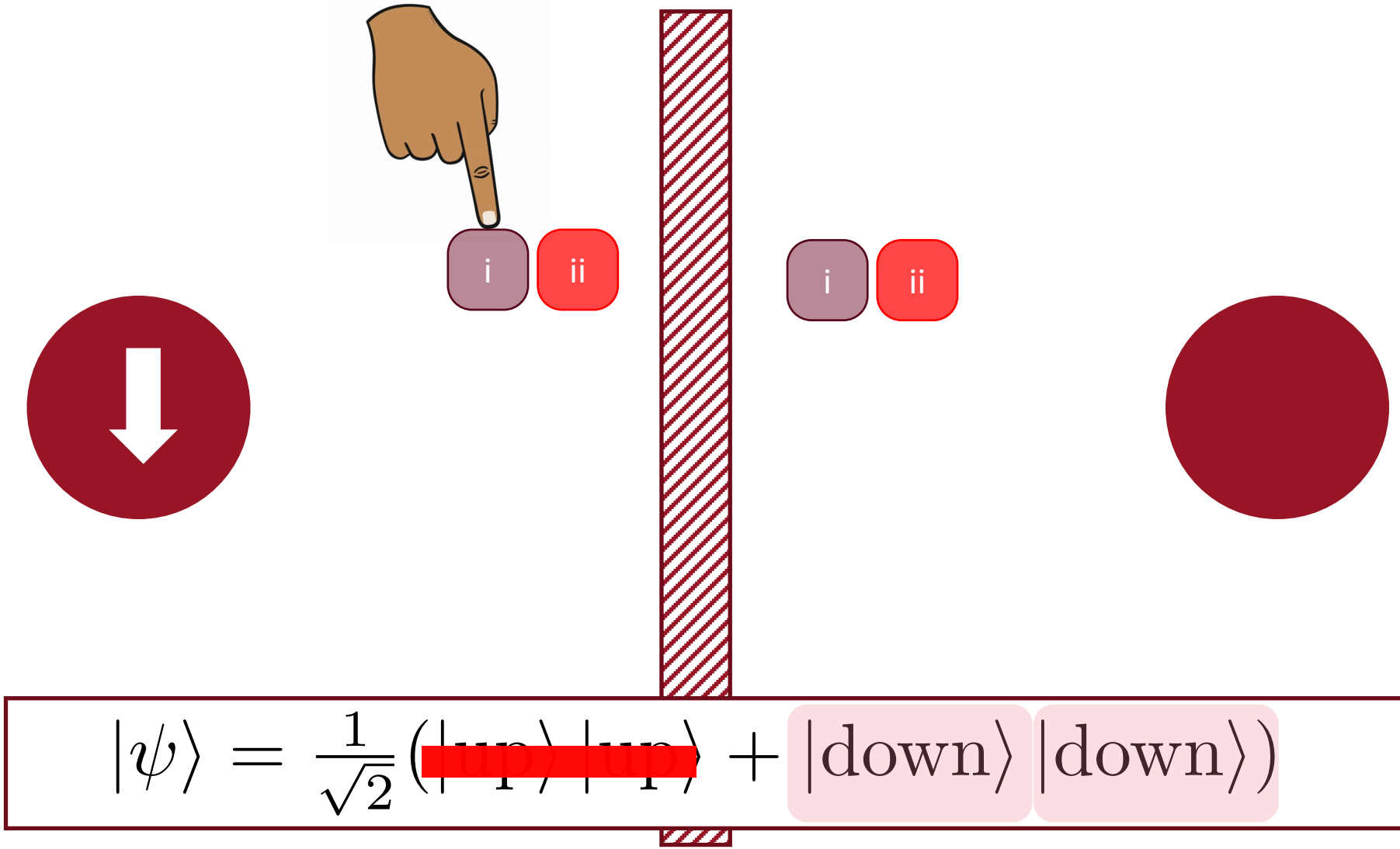
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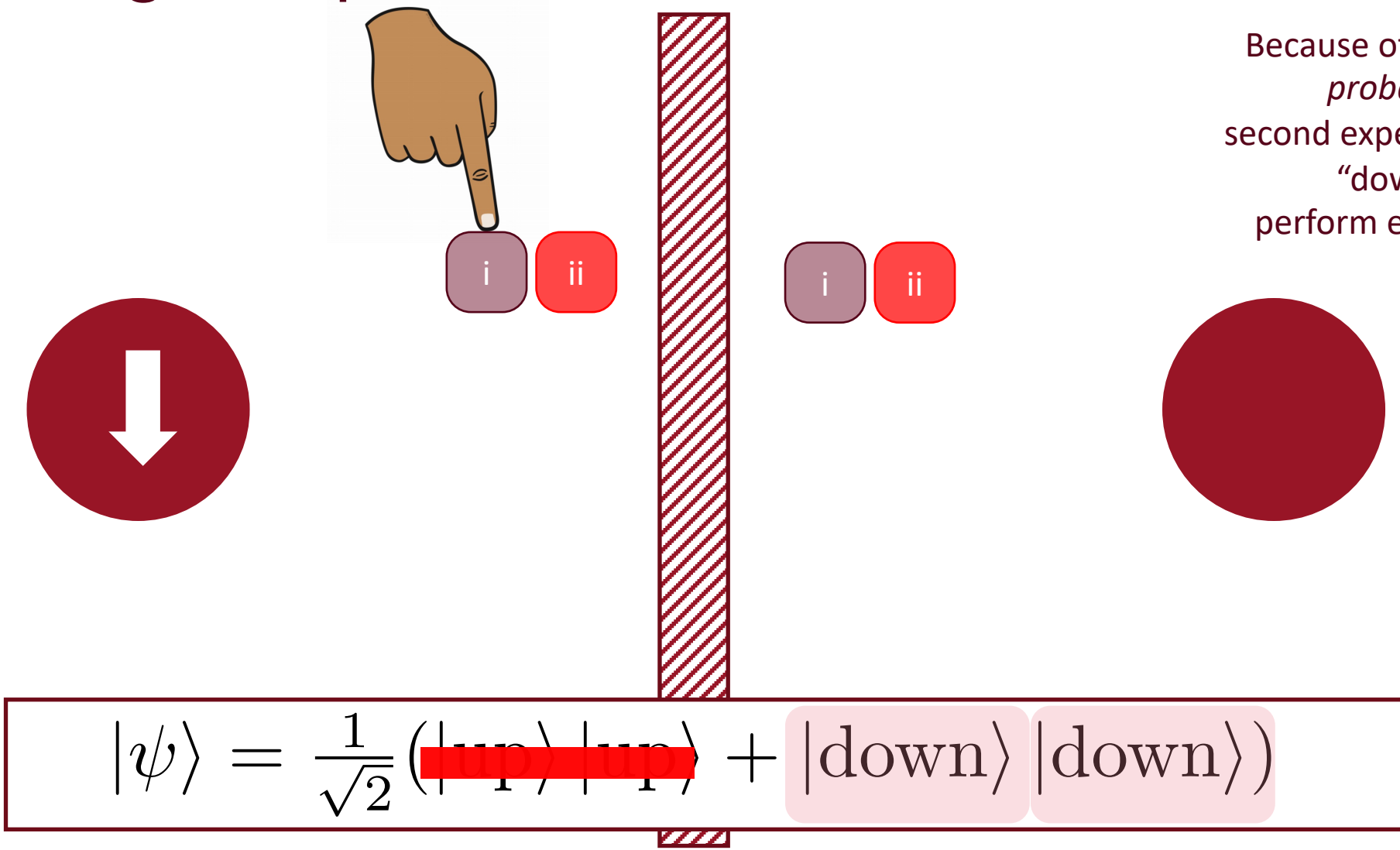
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EPR thought experiment



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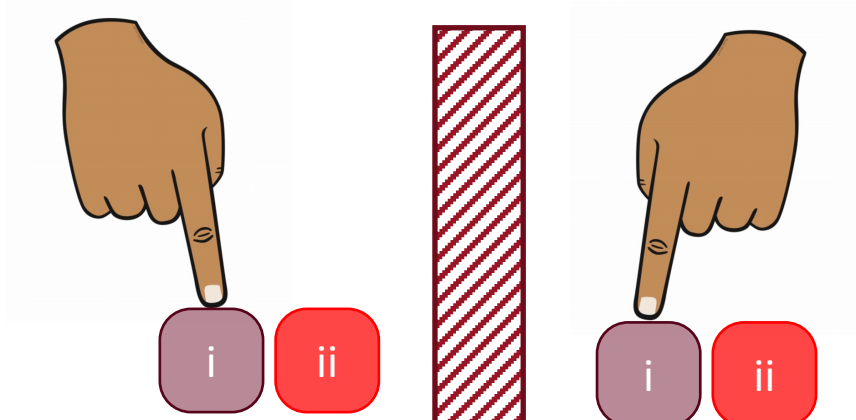
EPR thought experiment



Because of the state, the *probability* that the second experimenter sees "down" when they perform experiment 1 is certain.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle|\text{up}\rangle + |\text{down}\rangle|\text{down}\rangle)$$

EPR thought experiment



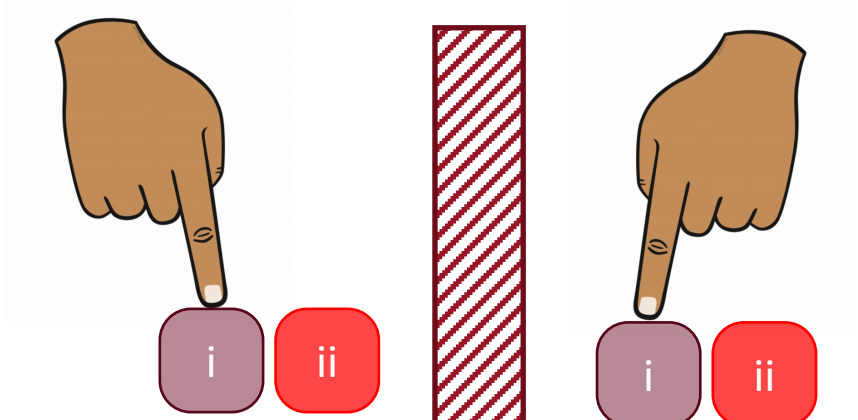
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$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle|\text{up}\rangle + |\text{down}\rangle|\text{down}\rangle)$$

EPR thought experiment



In quantum theory, the probability is computed as a function of the experiments (measurements) and the state via the Born rule

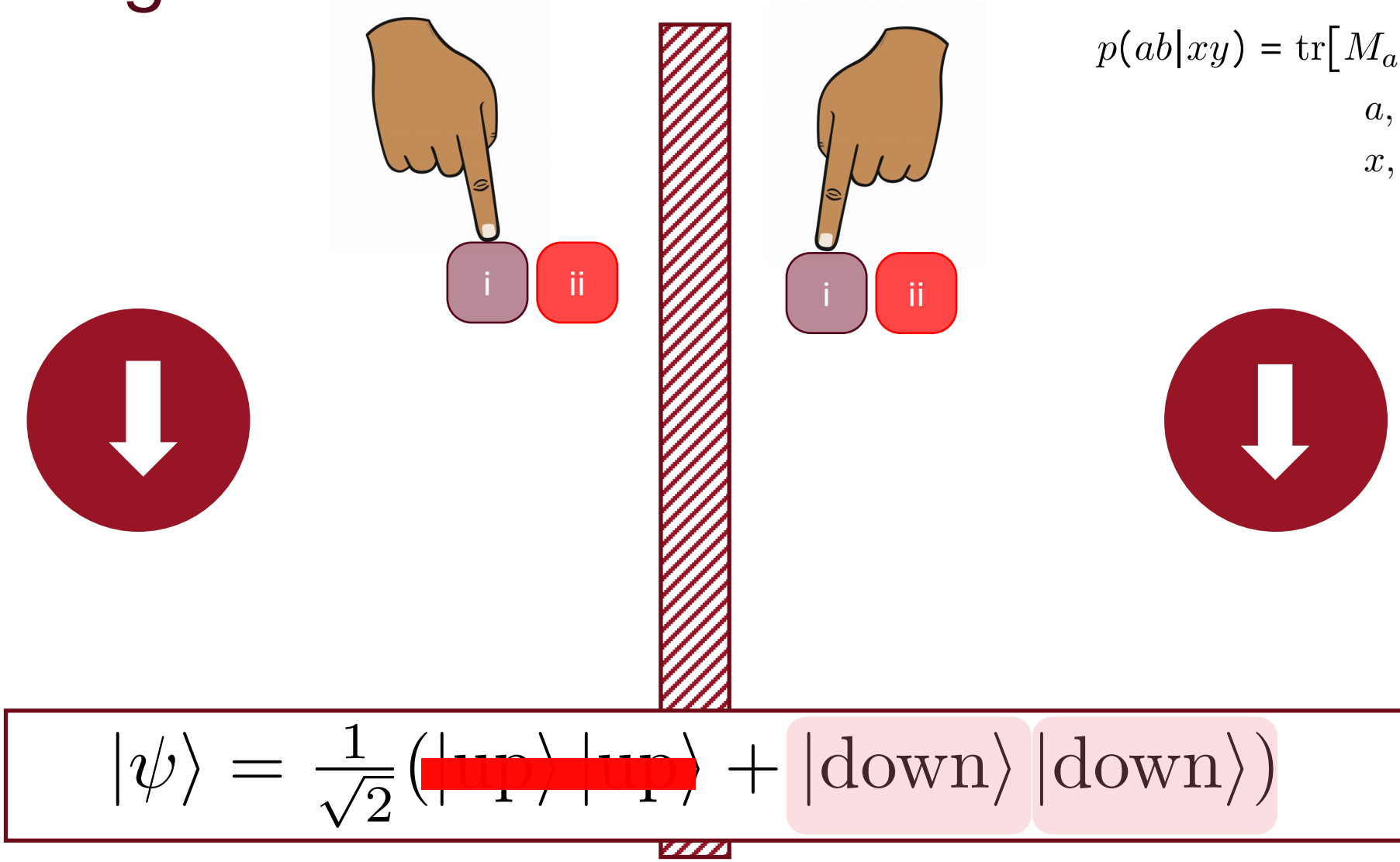
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EPR thought experiment

$$p(ab|xy) = \mathrm{tr}\left[M_{a|x} \otimes M_{b|y}\, \rho_{1,2}\right]$$

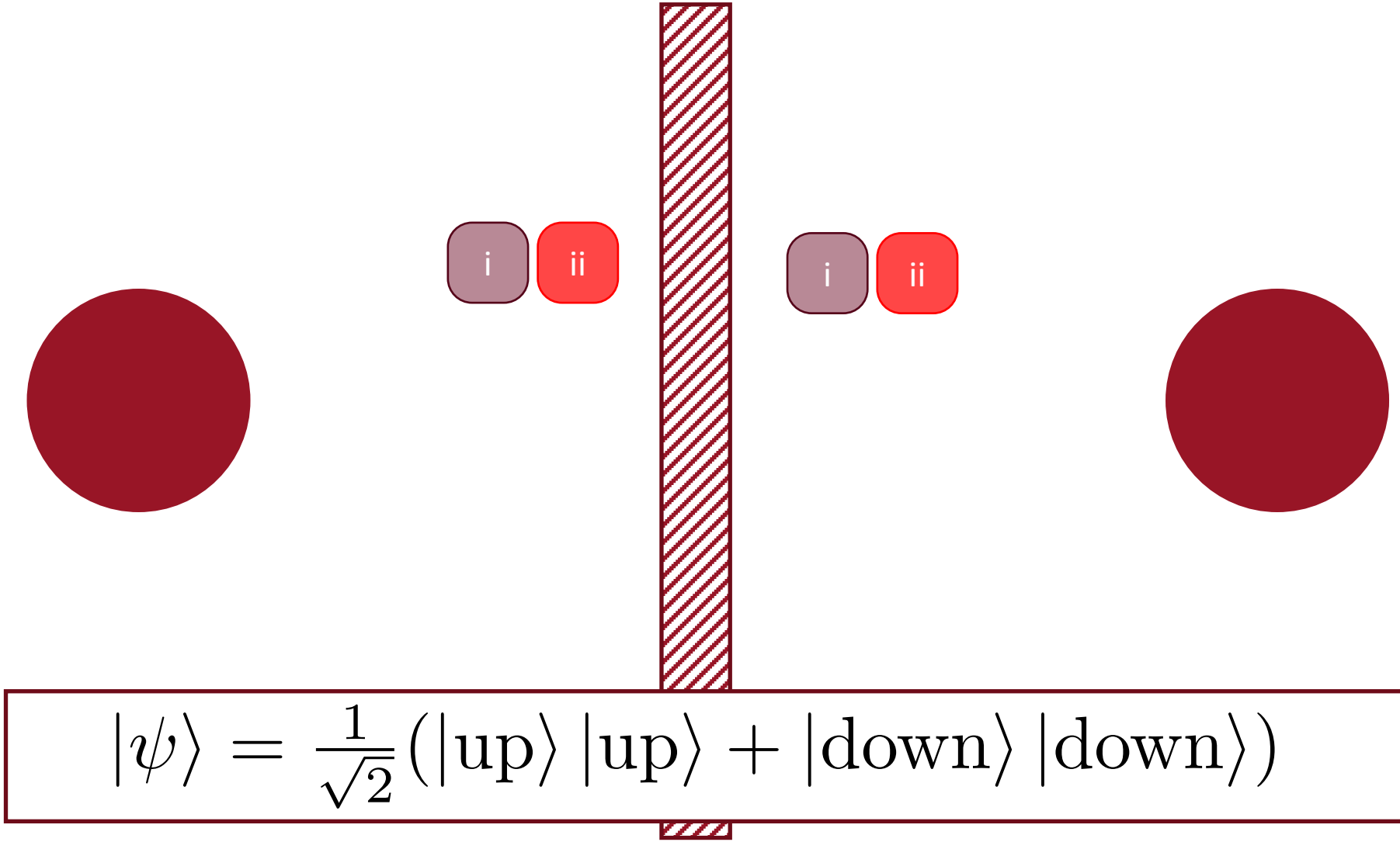
$$a, b \in \{\mathrm{up}, \mathrm{down}\}$$

$$x, y \in \{\mathrm{i}, \mathrm{ii}\}$$



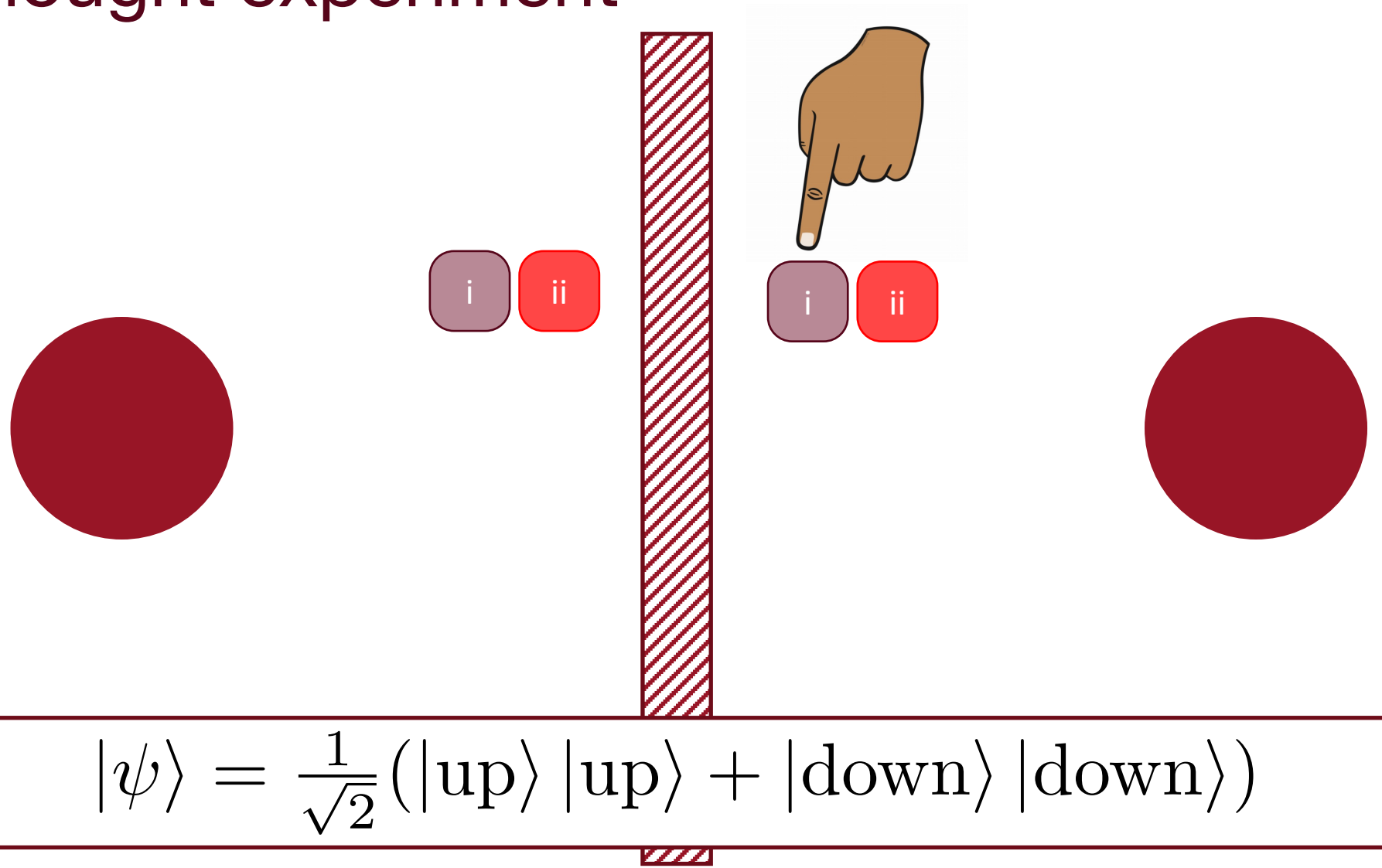
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EPR thought experiment



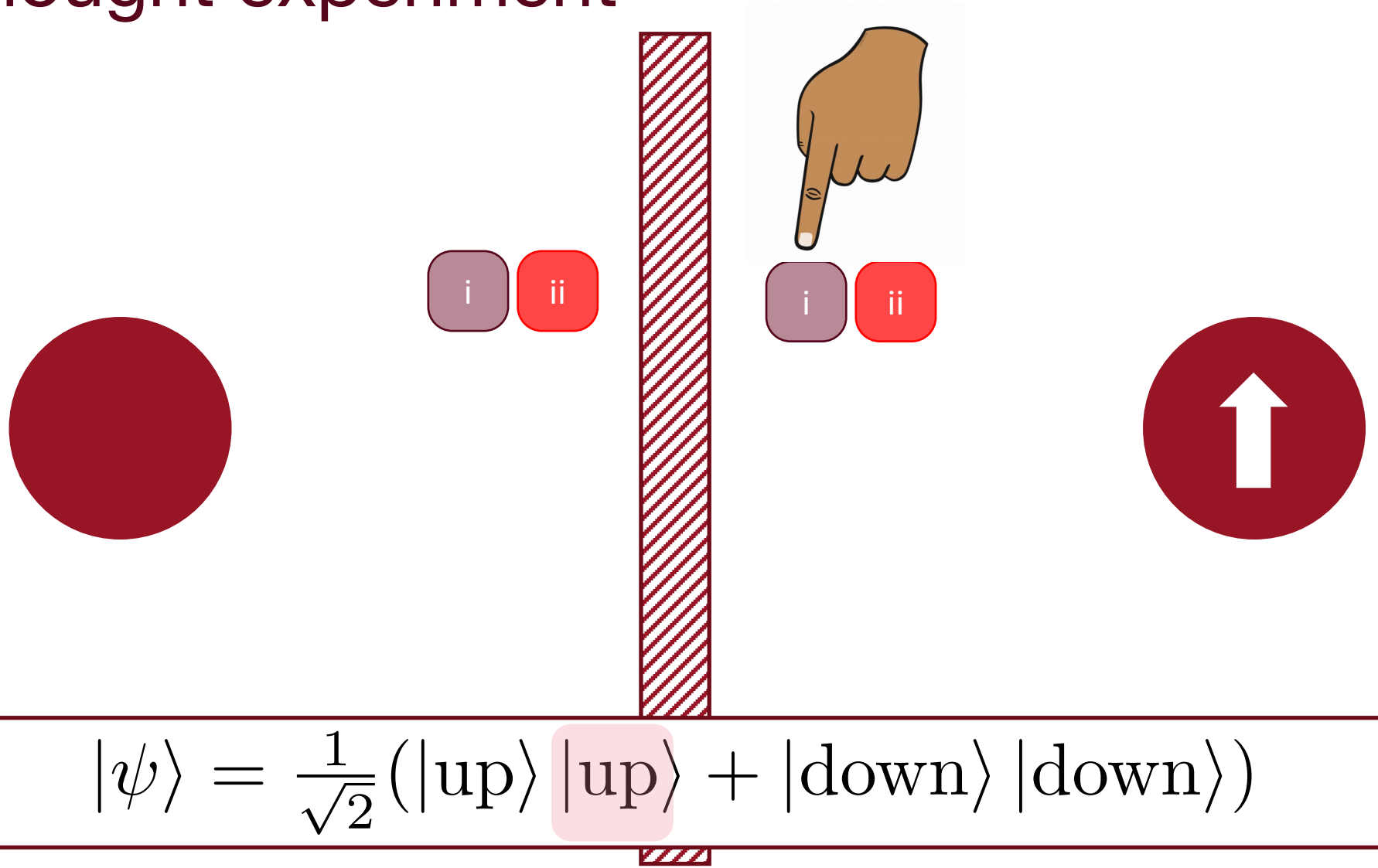
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EPR thought experiment



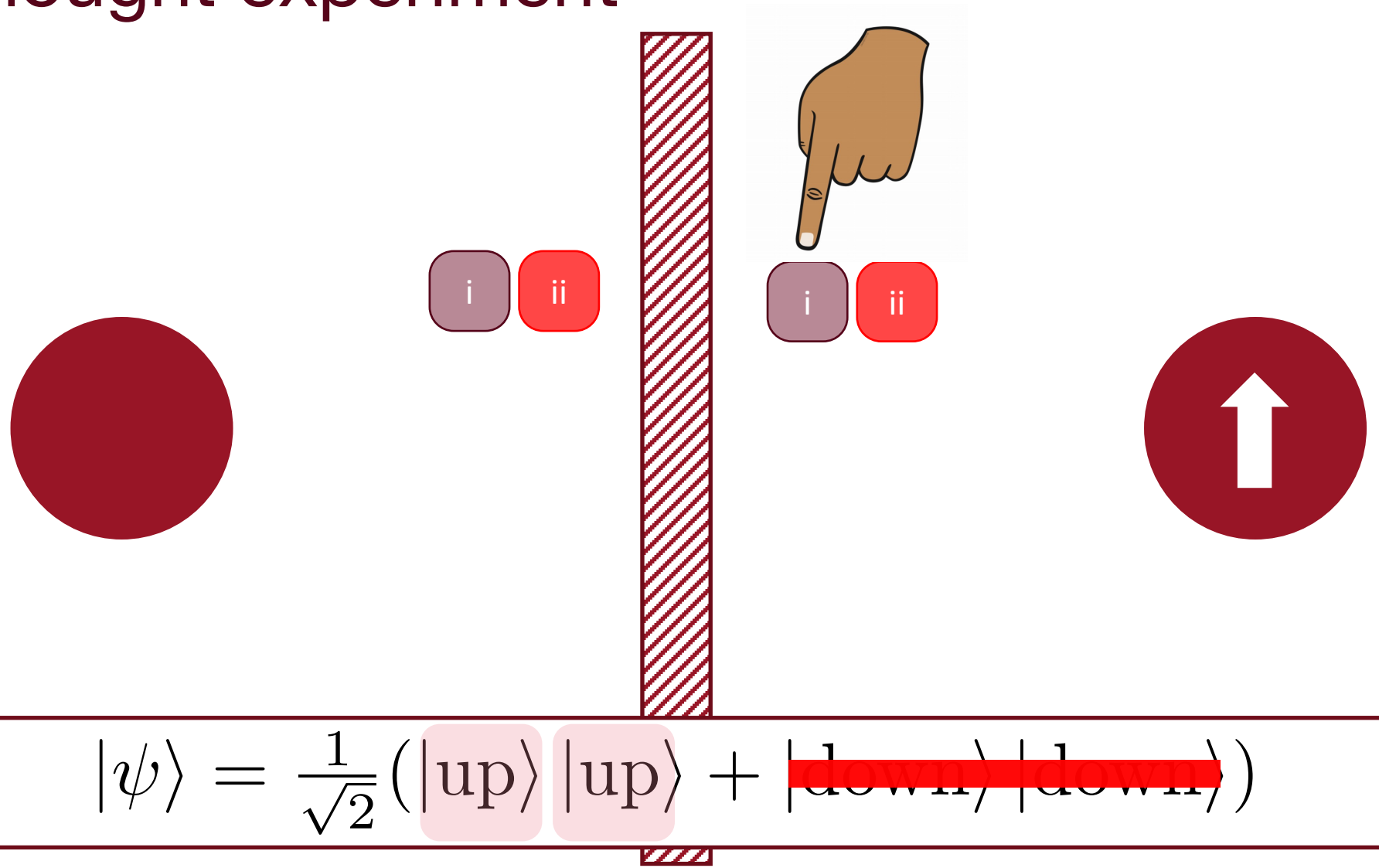
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EPR thought experiment



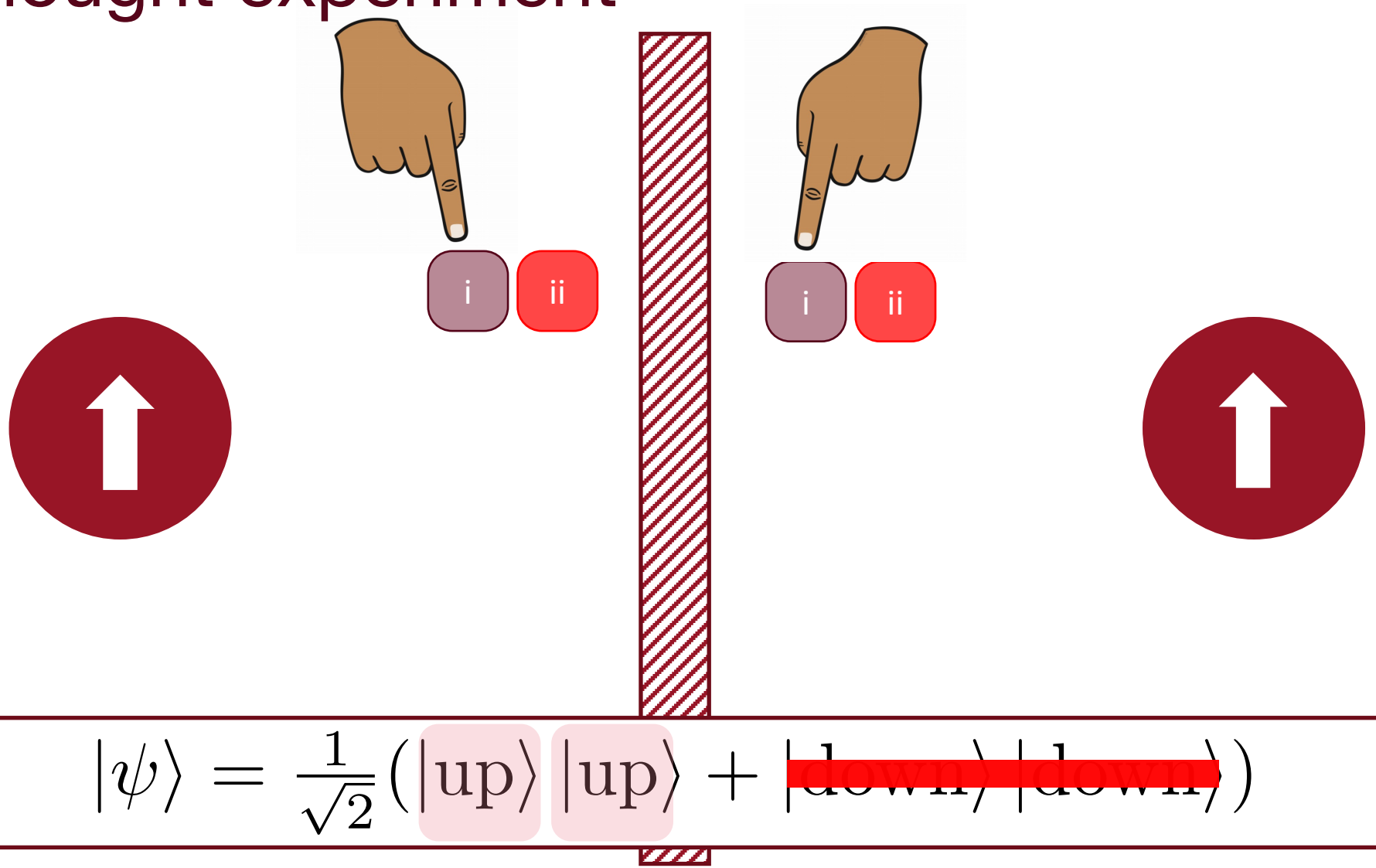
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EPR thought experiment



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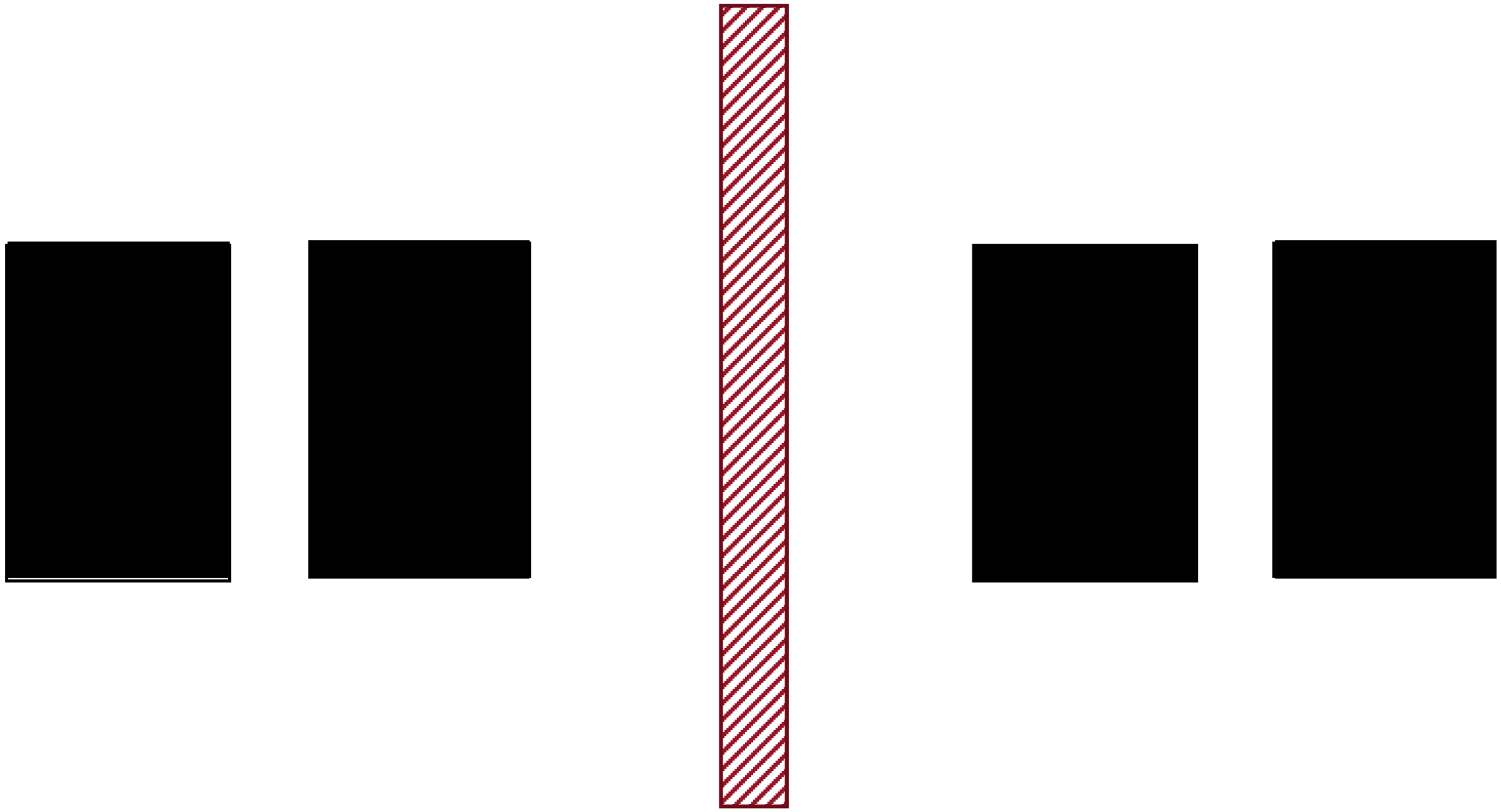
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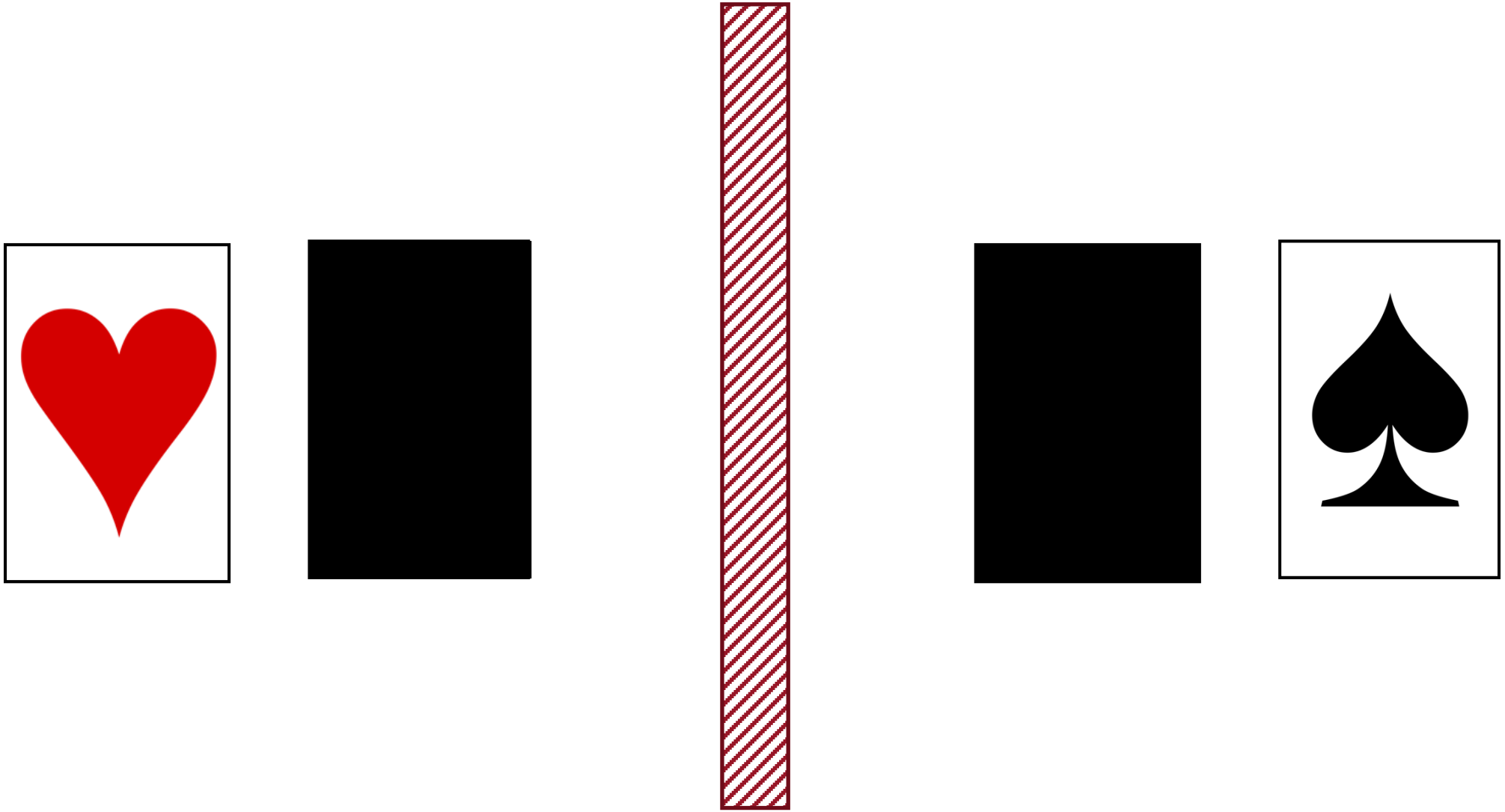
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Let’s get correlated.

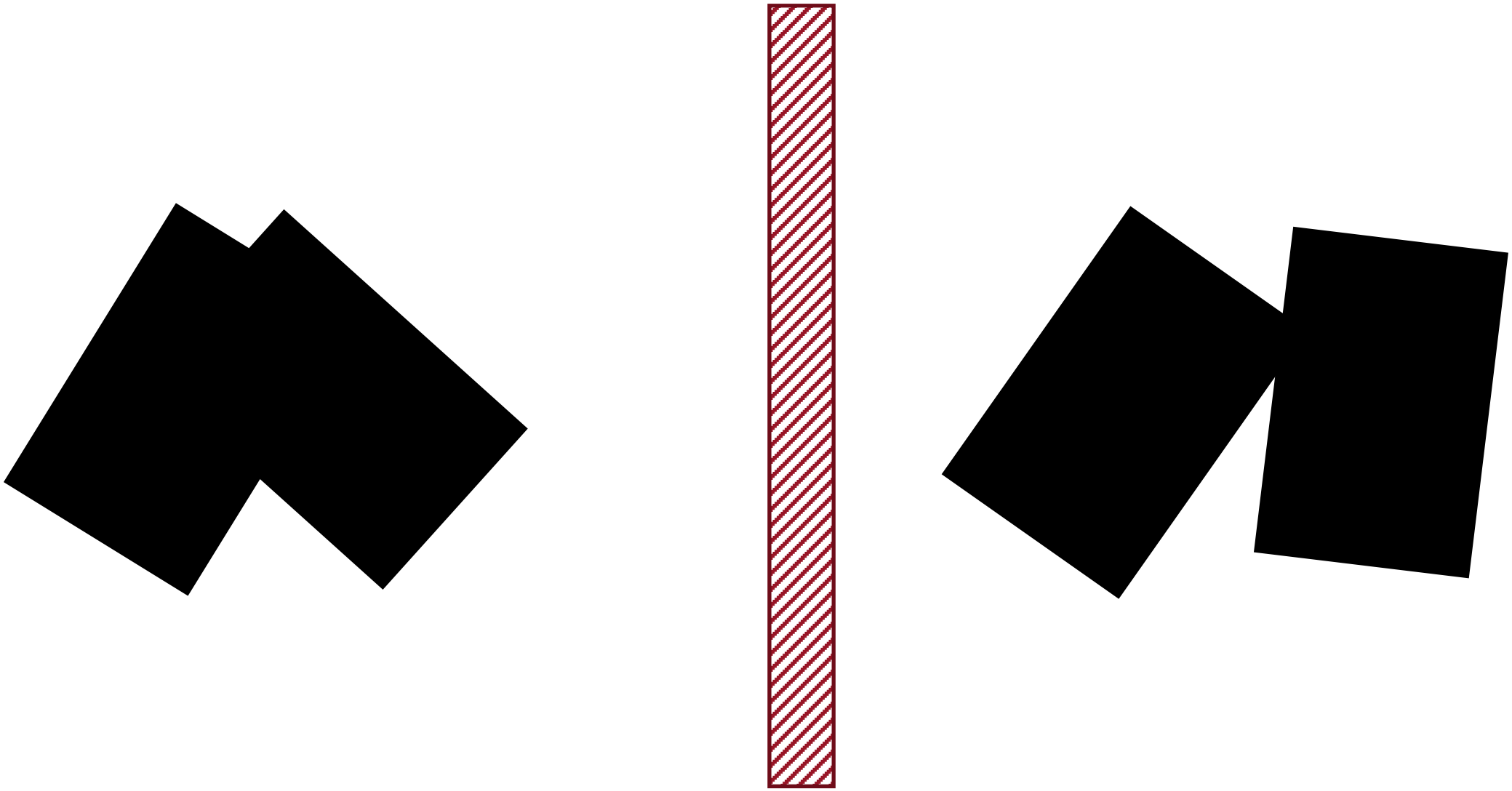
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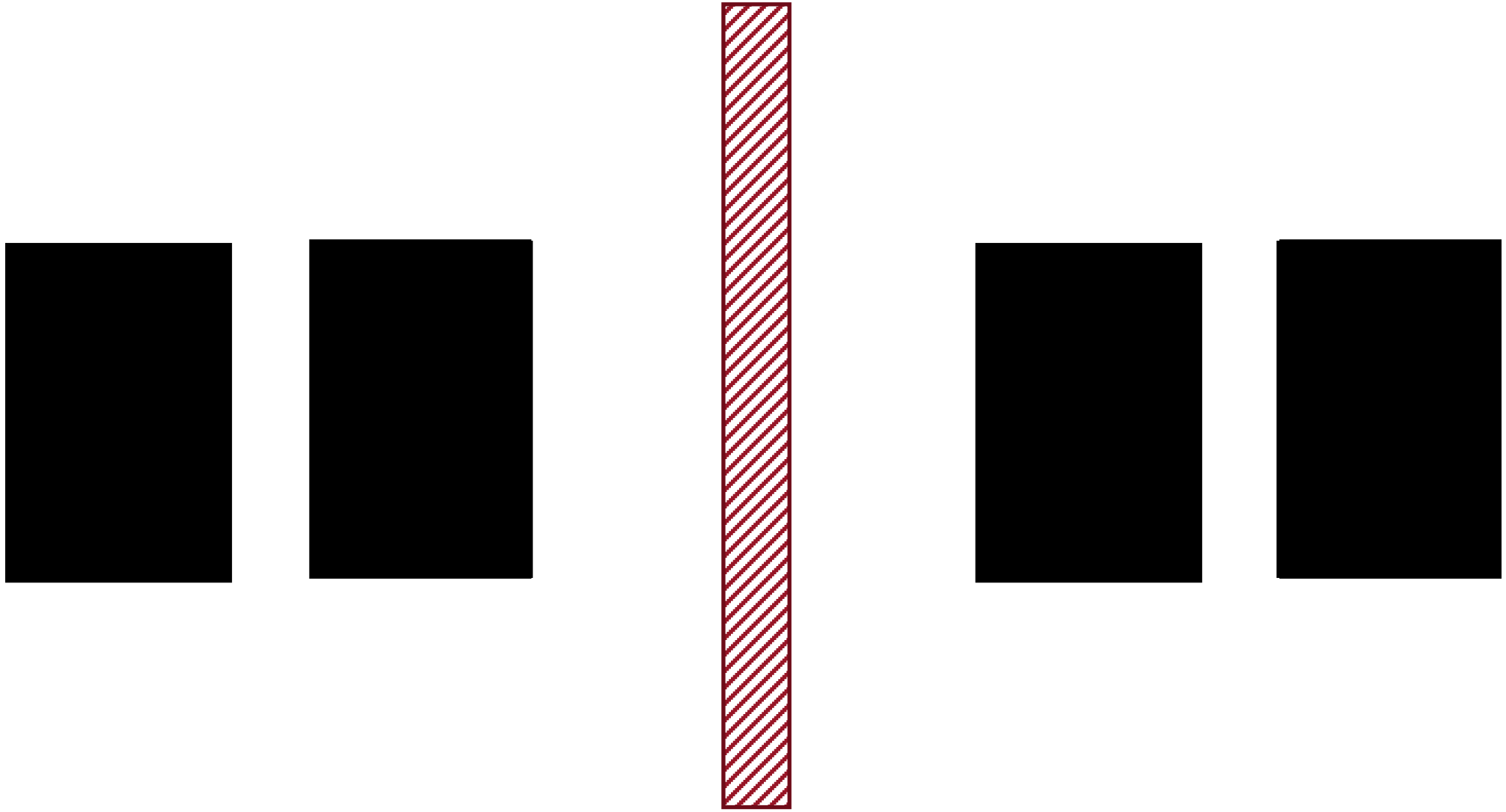
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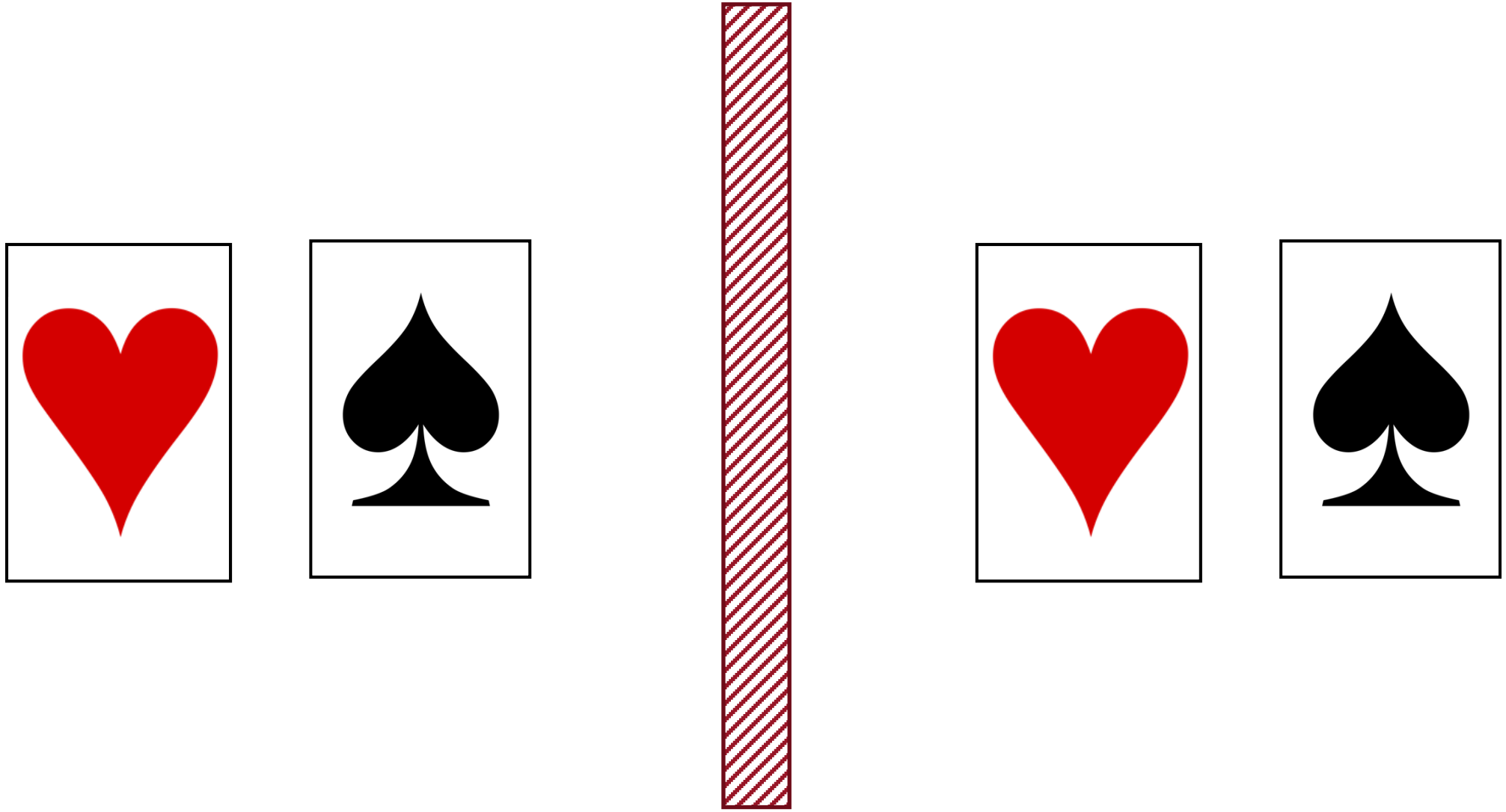
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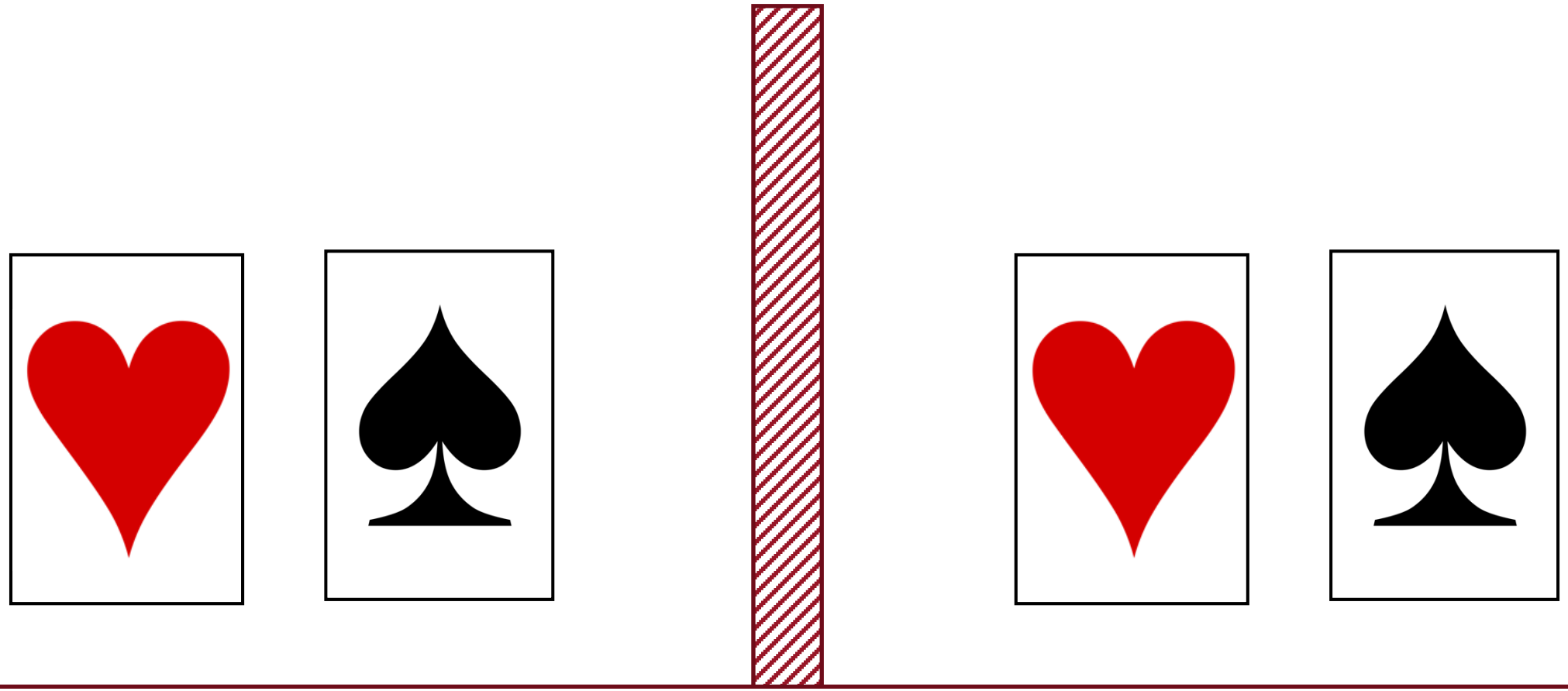
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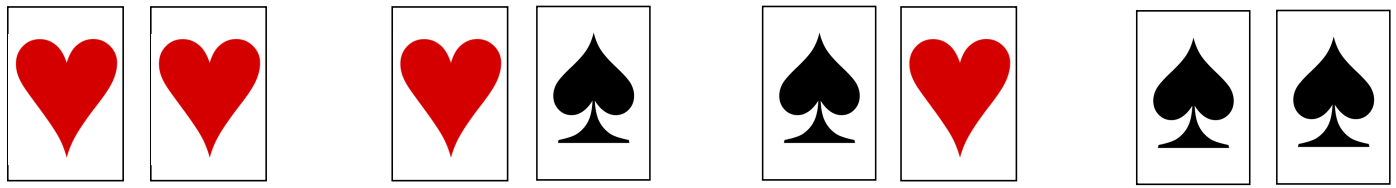
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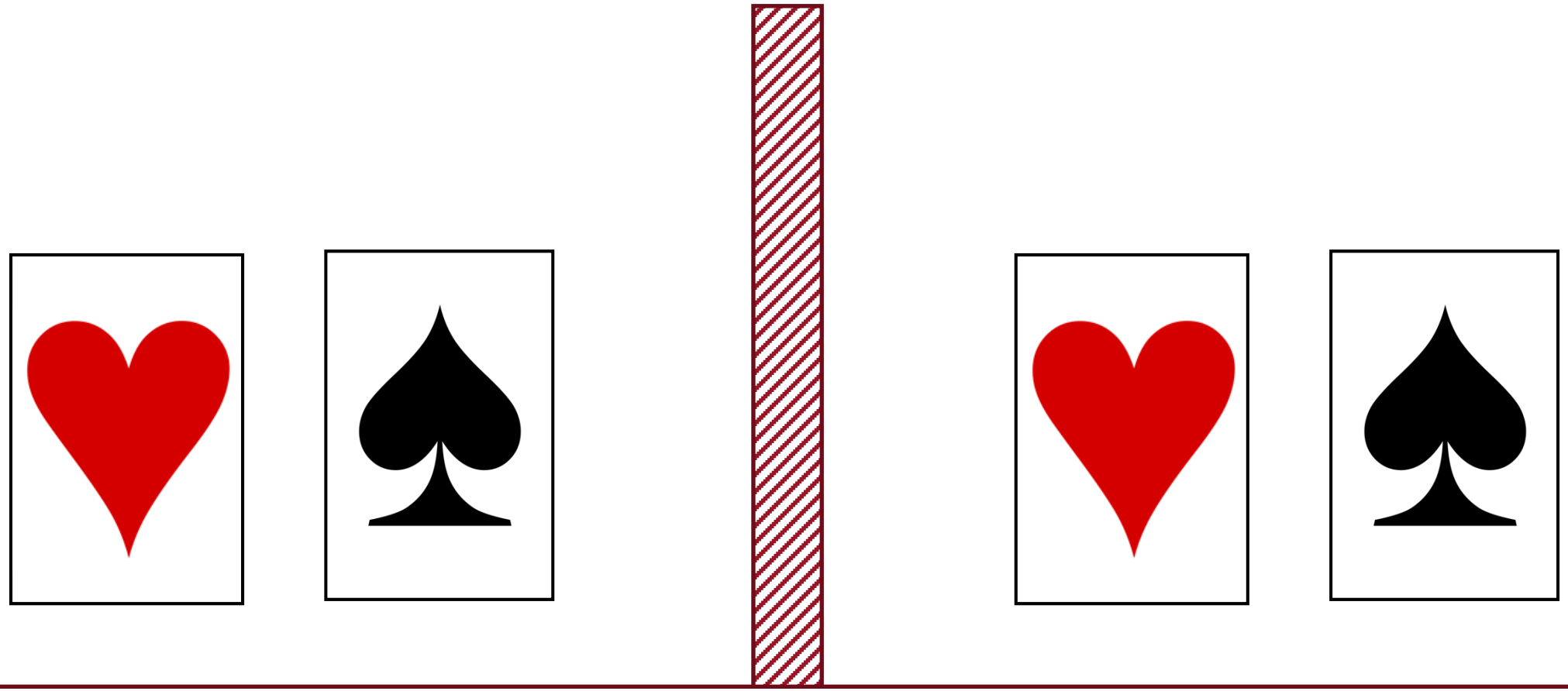
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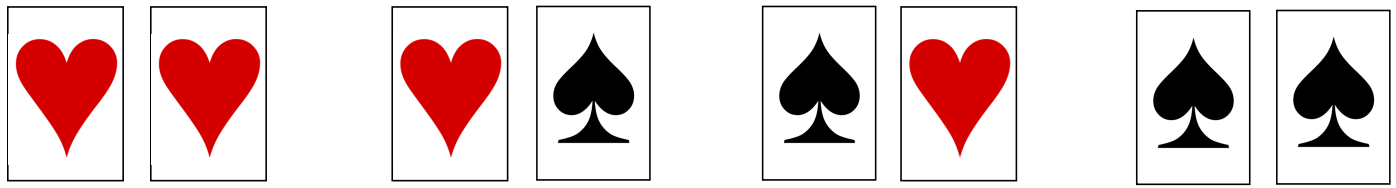
Possible outcomes:



Let's get (not) correlated.

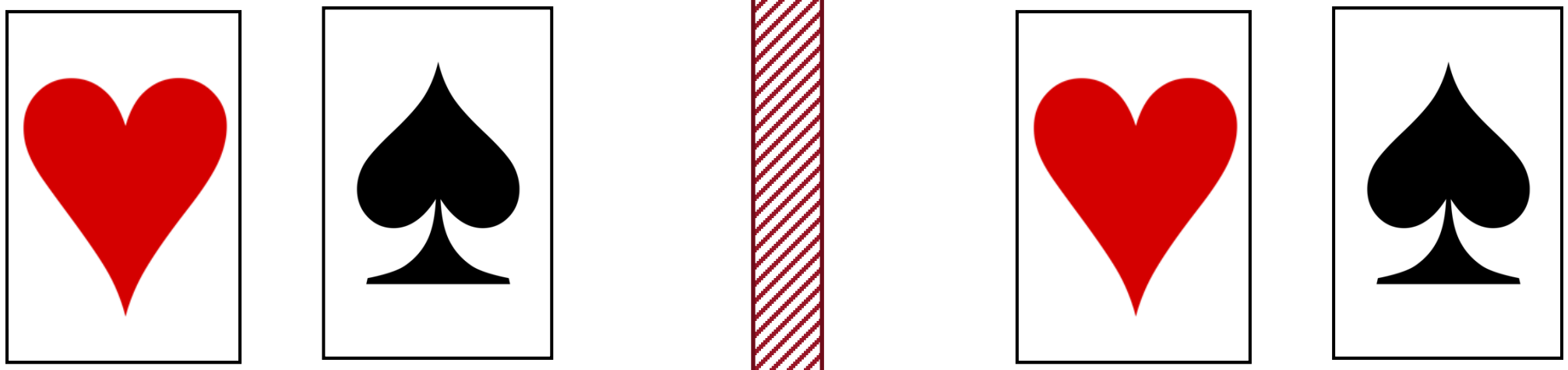


Possible outcomes:

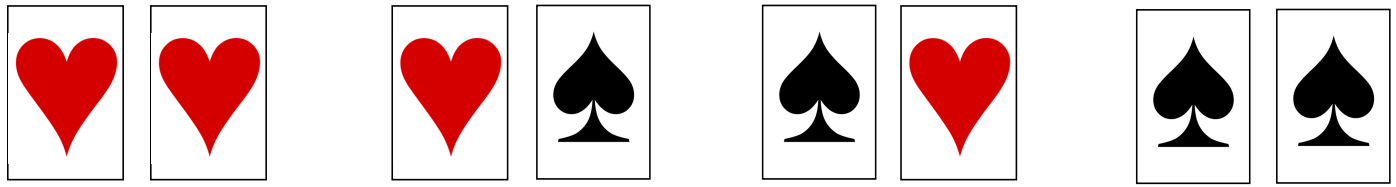


Let's get (not) correlated.

Does the result of player 2 depends on player 1?



Possible outcomes:

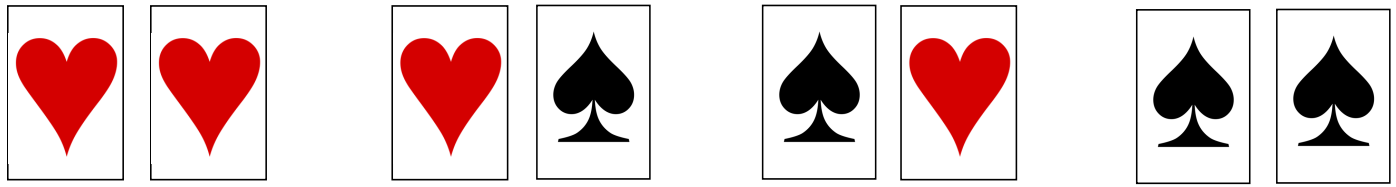


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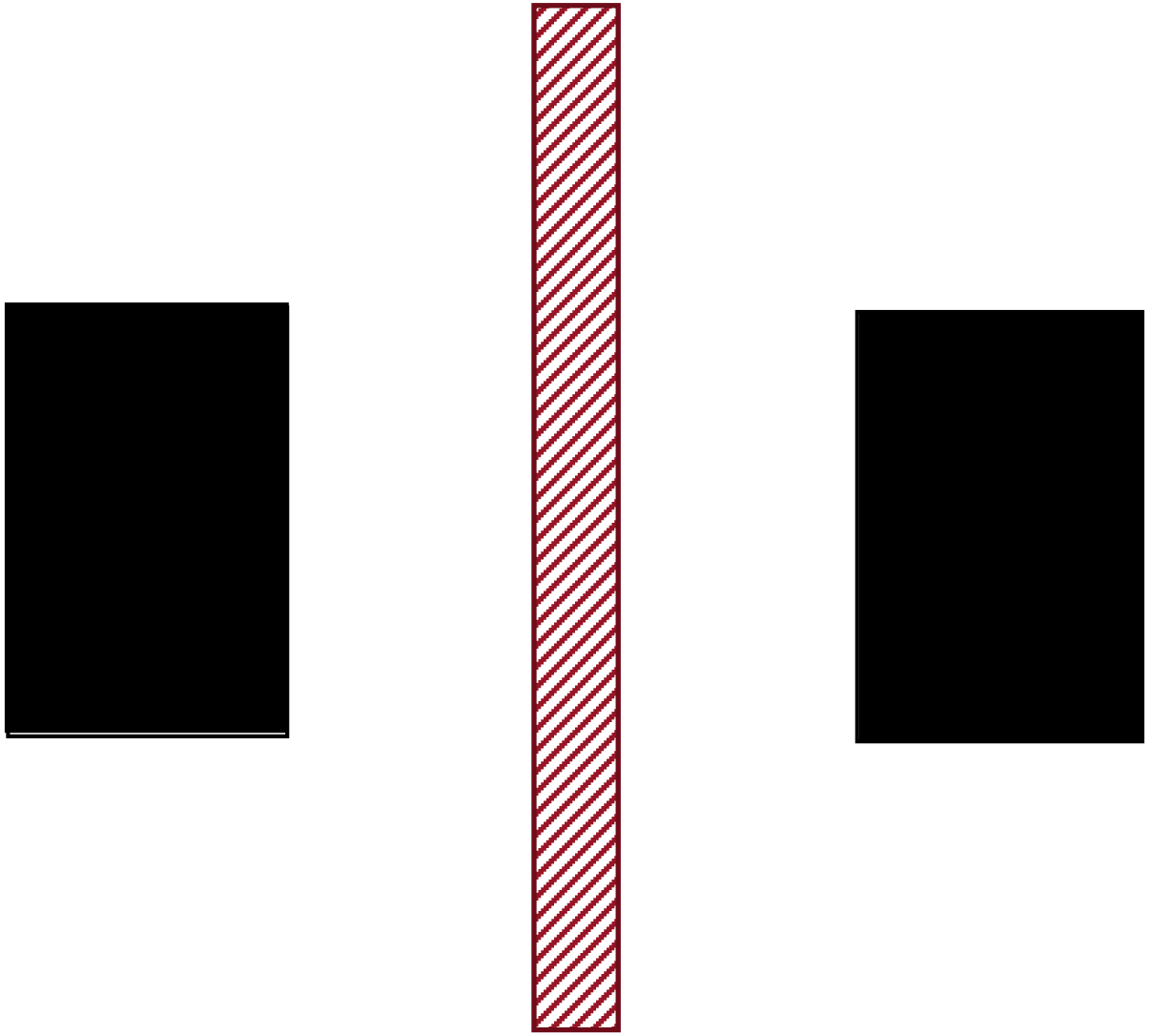
$$\rho_{1,2} = \frac{1}{4}\,|\heartsuit\heartsuit\rangle\langle\heartsuit\heartsuit| + \frac{1}{4}\,|\heartsuit\spadesuit\rangle\langle\heartsuit\spadesuit| + \frac{1}{4}\,|\spadesuit\heartsuit\rangle\langle\spadesuit\heartsuit| + \frac{1}{4}\,|\spadesuit\spadesuit\rangle\langle\spadesuit\spadesuit|$$

$$= \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

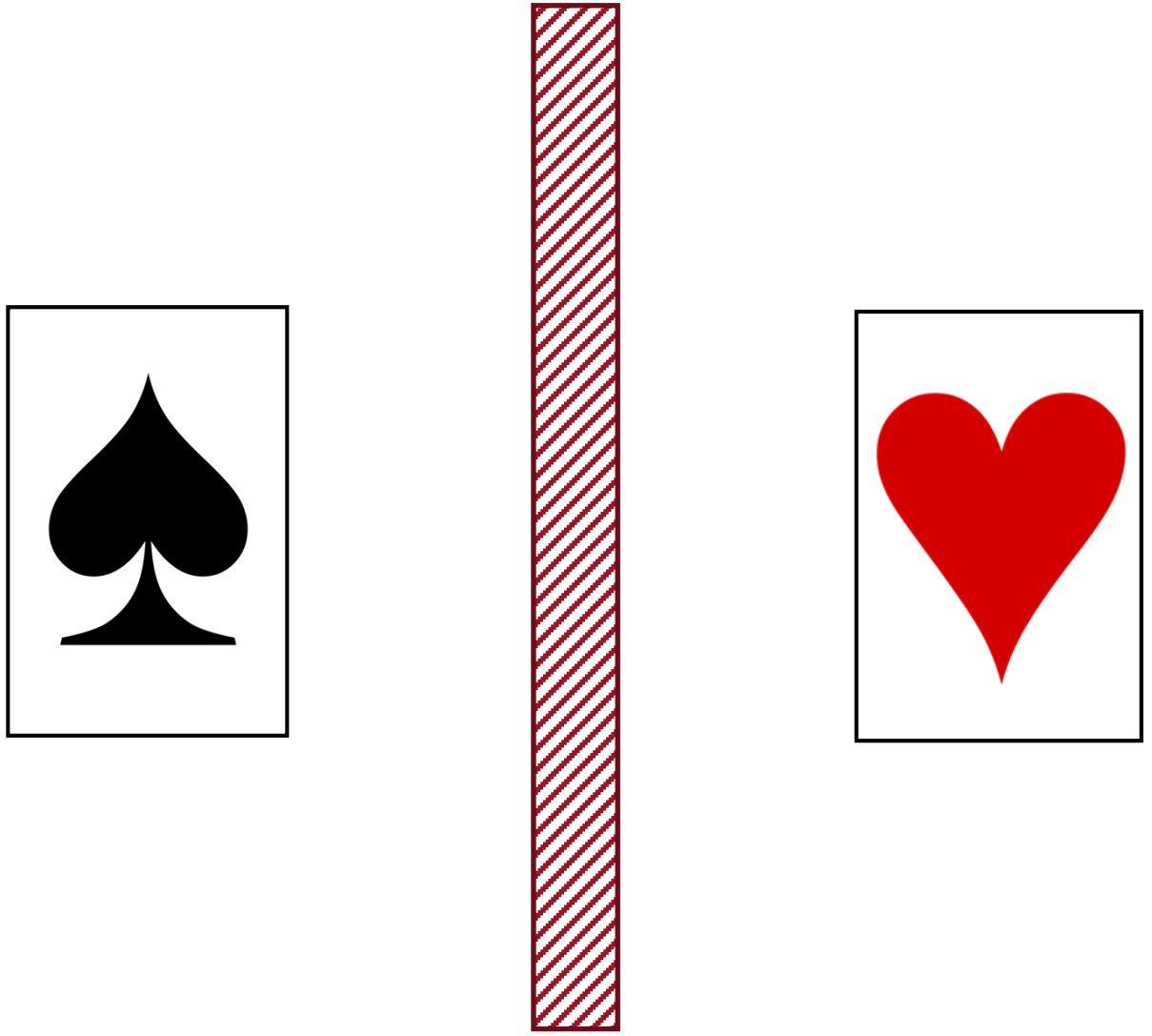
Possible outcomes:



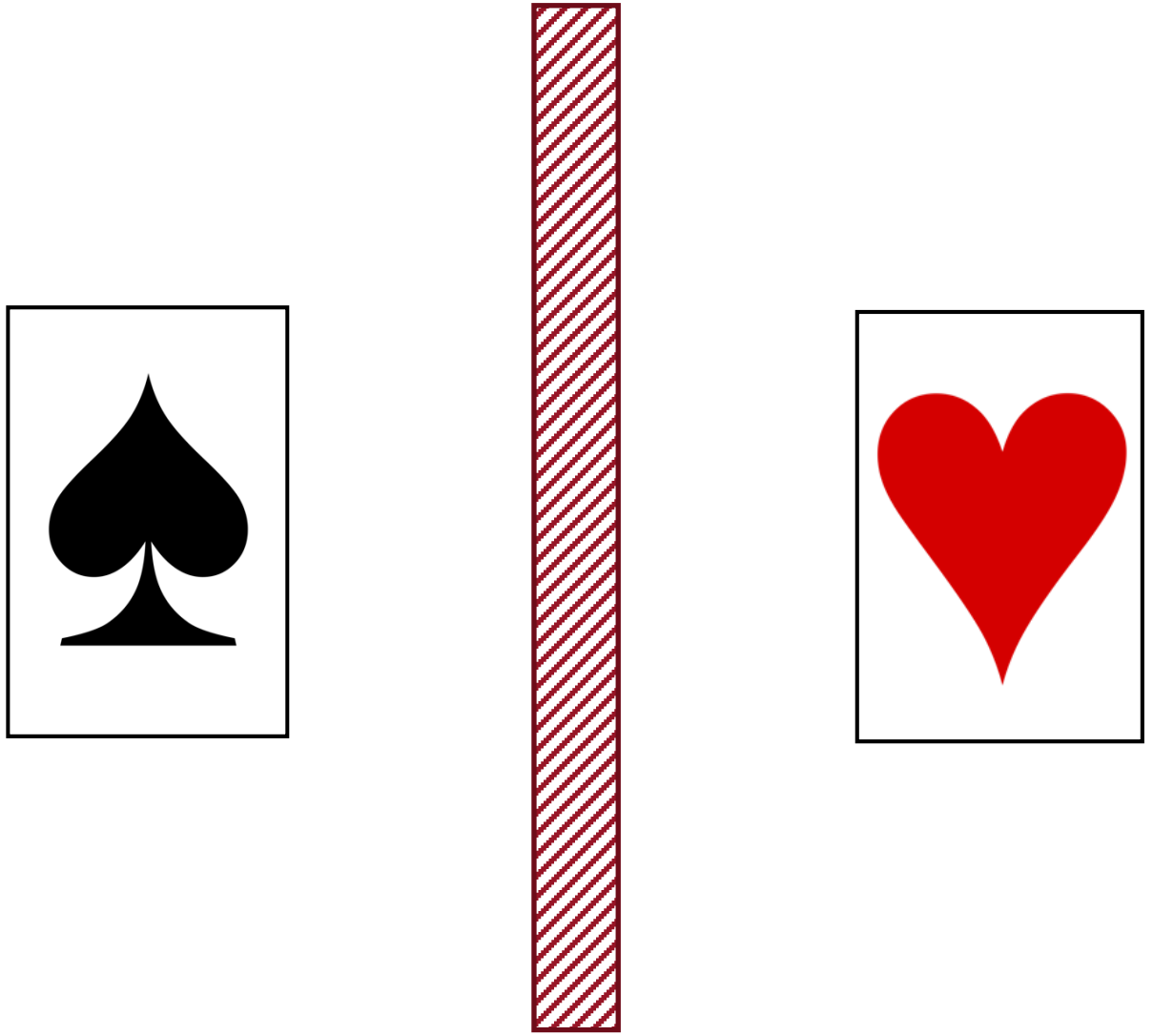
Let’s get correlated.



Let’s get correlated.

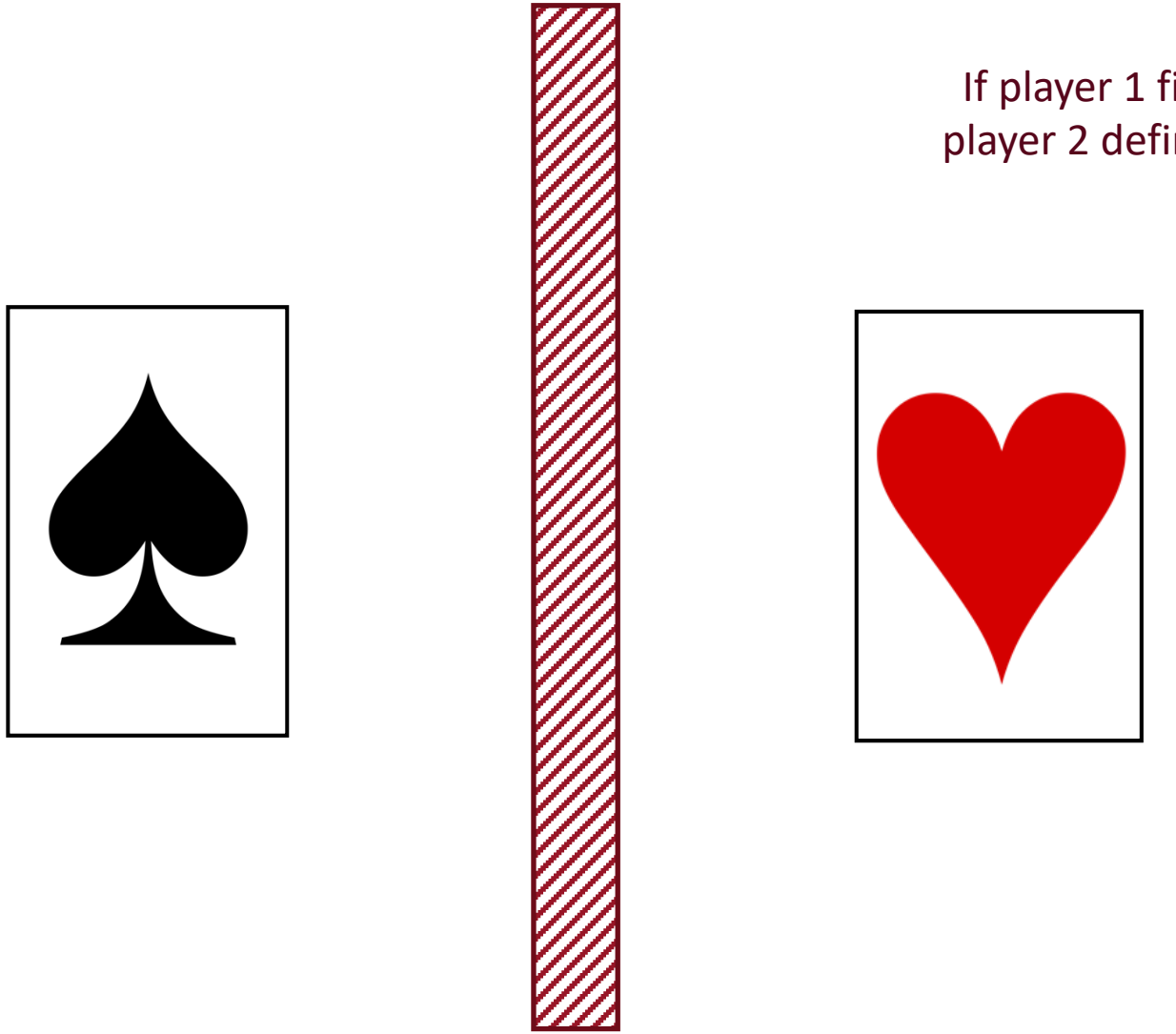


Let's get (completely) correlated.



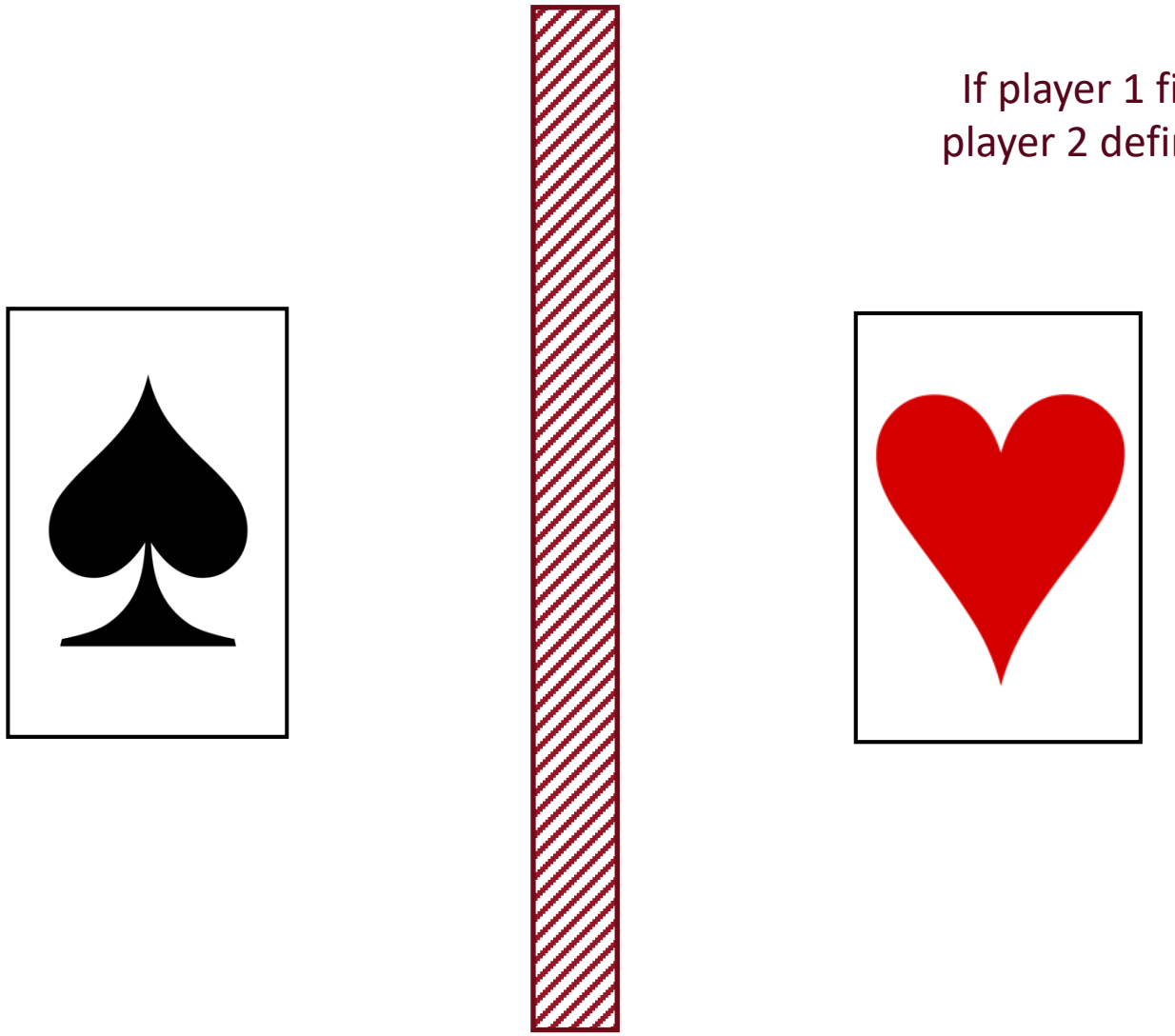
Example 2.

Let's get (completely) correlated.



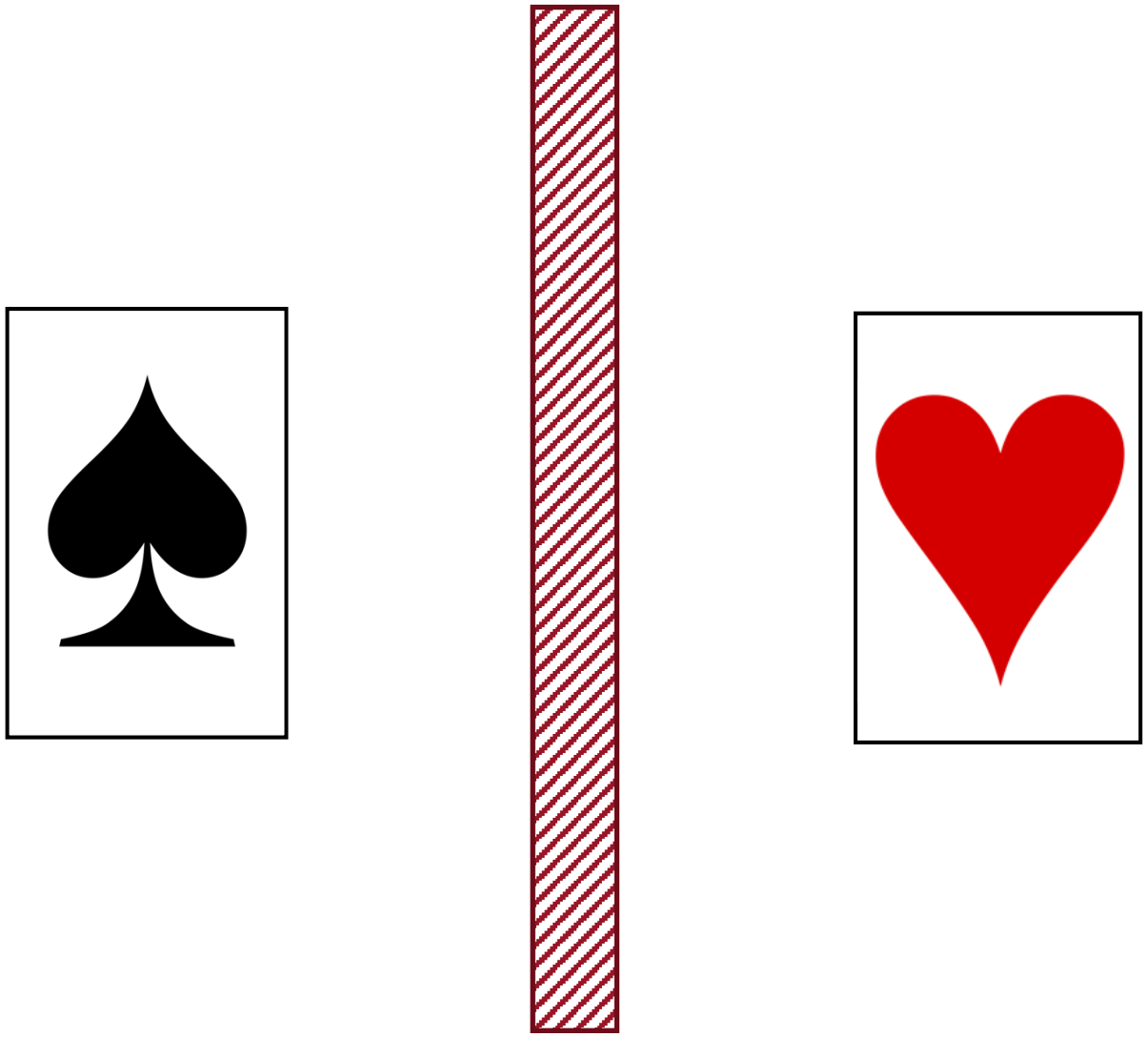
If player 1 finds their card is black, then player 2 definitely finds that their card is the opposite colour – red.

Let's get (completely) anti-correlated.



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Let's get (completely) anti-correlated.



If player 1 finds their card is black, then player 2 definitely finds that their card is the opposite colour – red.

$$\rho_{1,2} = |\spadesuit\heartsuit\rangle\langle\spadesuit\heartsuit|$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Let's get correlated.

$$\rho_{1,2} = \frac{1}{2}\,|\heartsuit\heartsuit\rangle\langle\heartsuit\heartsuit| + \frac{1}{2}\,|\spadesuit\spadesuit\rangle\langle\spadesuit\spadesuit|$$

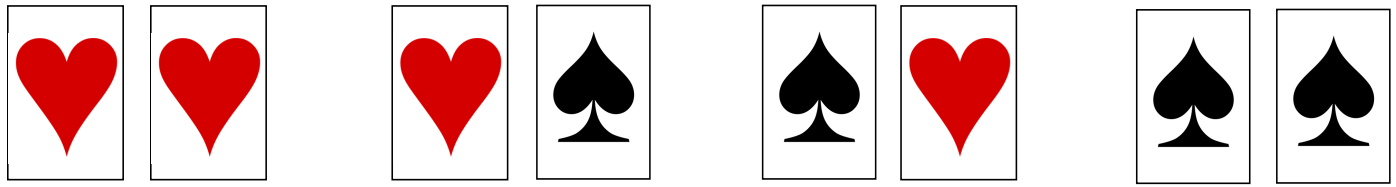
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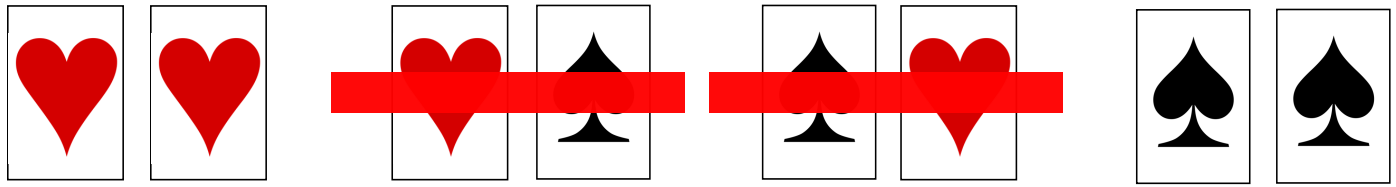


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Possible outcomes:



Let's get (classically) correlated.



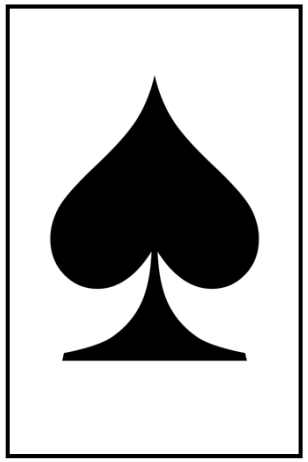




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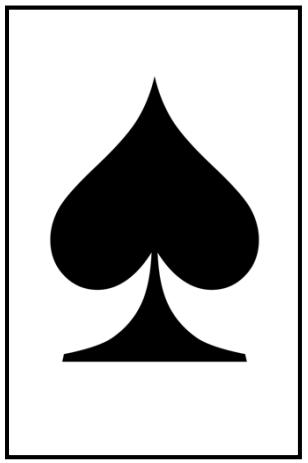
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Let's get (classically) correlated.



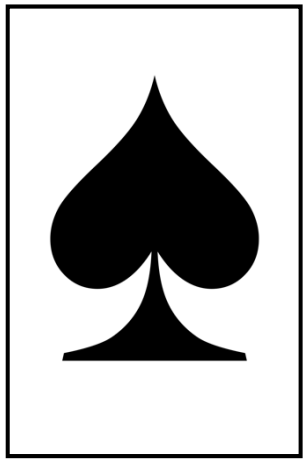
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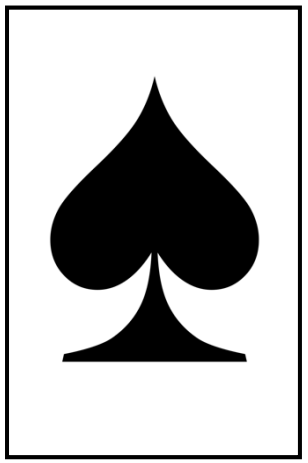




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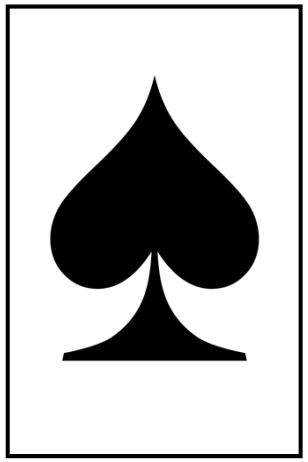
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Let's get (classically) correlated.



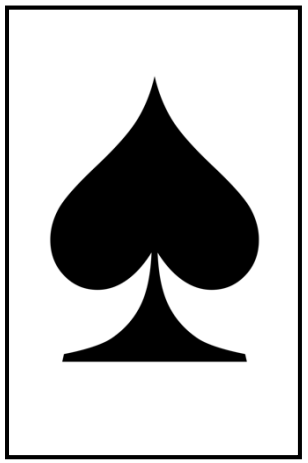


Once the first person has performed their experiment, the outcome for the second person is **certain.**

$$\rho_{1,2} = \tfrac{1}{2}\,|\heartsuit\heartsuit\rangle\langle\heartsuit\heartsuit| + \tfrac{1}{2}\,|\spadesuit\spadesuit\rangle\langle\spadesuit\spadesuit|$$

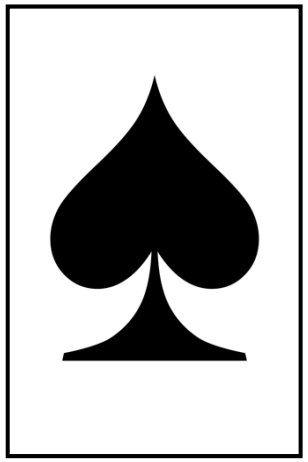
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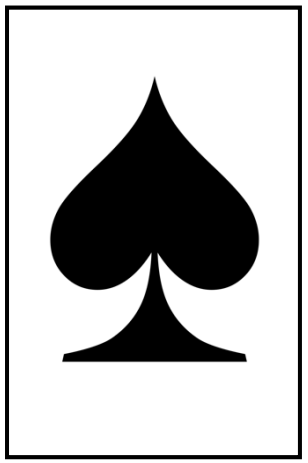


The measurement being made is colour: red/black?

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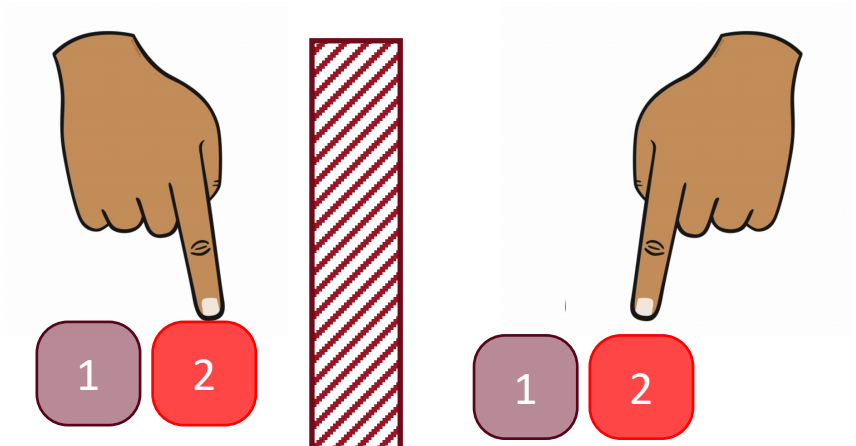
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Back to EPR

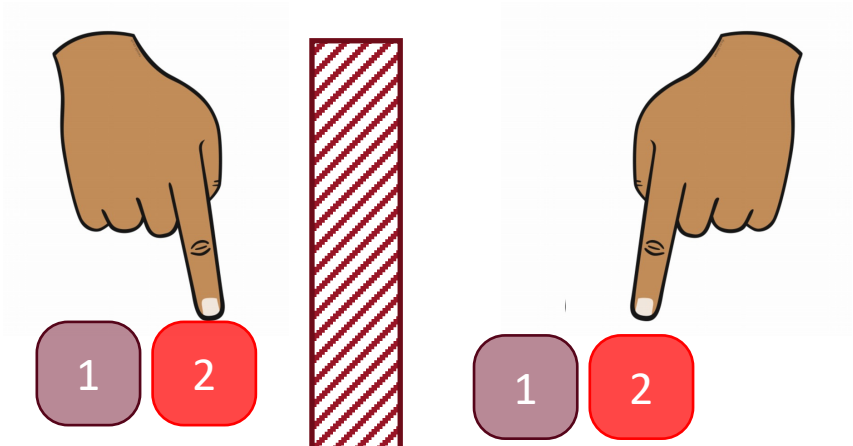
There are **two** experiments that exist such that when the first person has performed their measurement, the outcome for the second experimenter is certain.



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle\,|\text{up}\rangle + |\text{down}\rangle\,|\text{down}\rangle)$$

Back to EPR

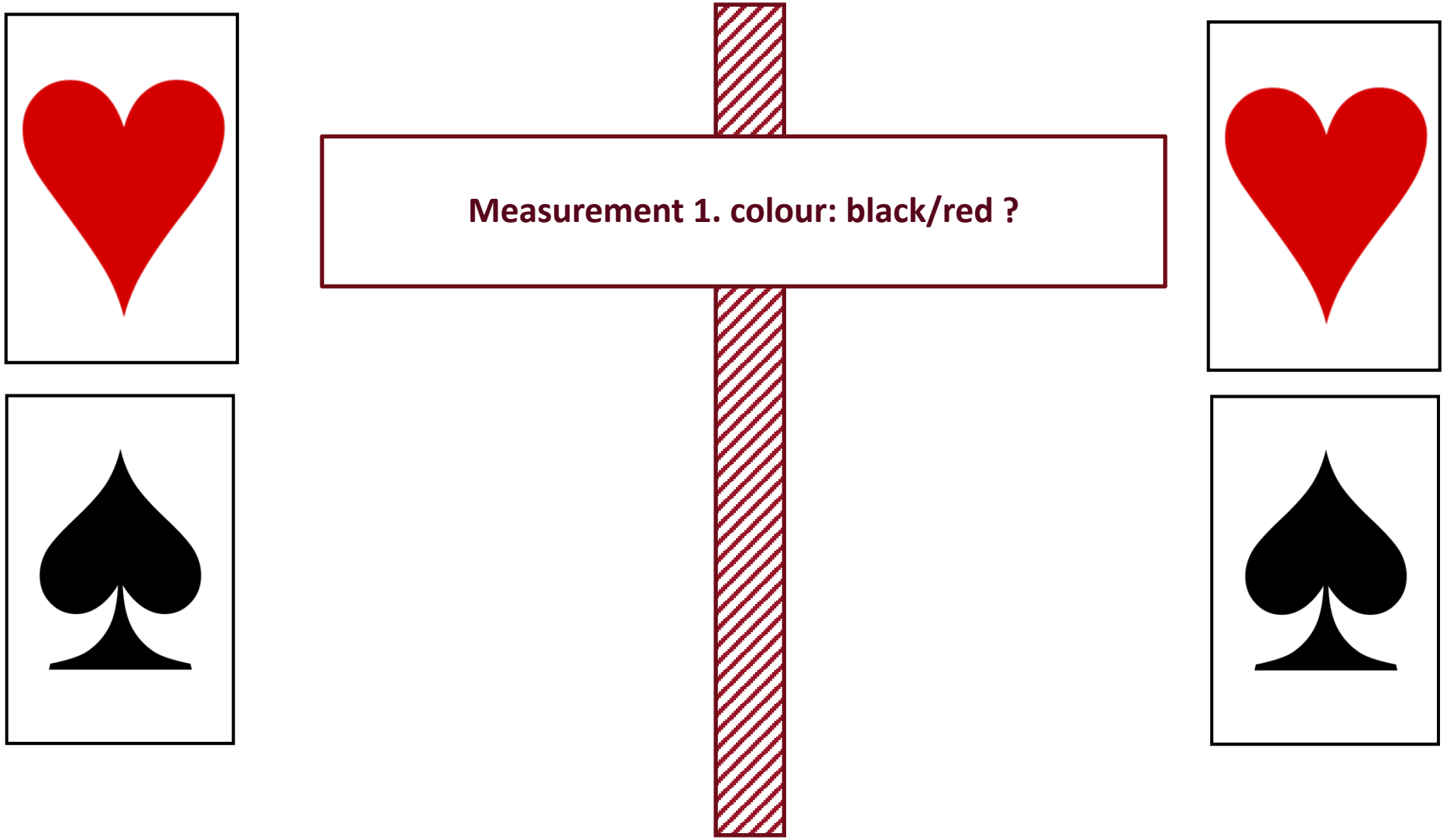
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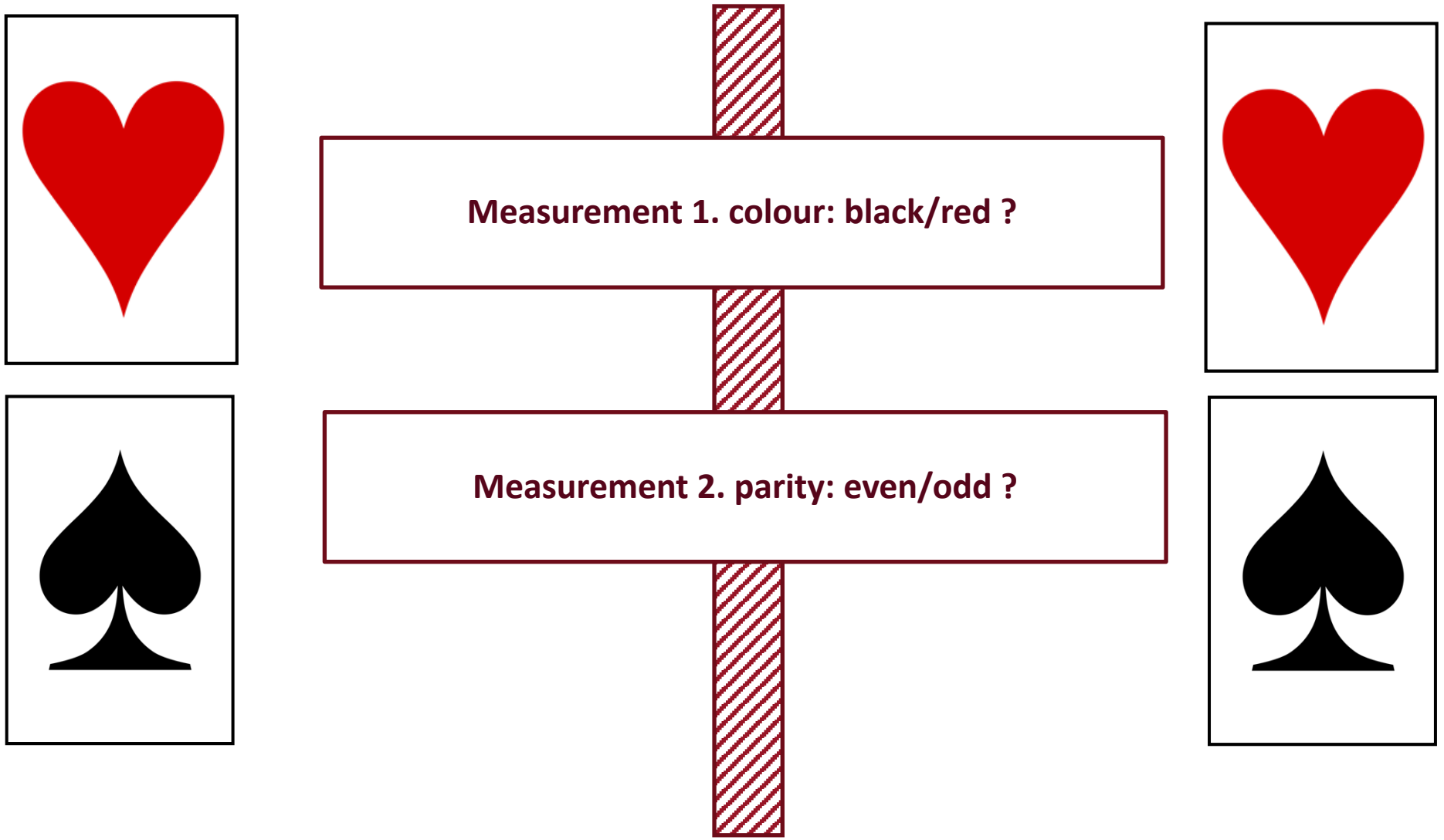
Example 3.

Classical versus quantum properties



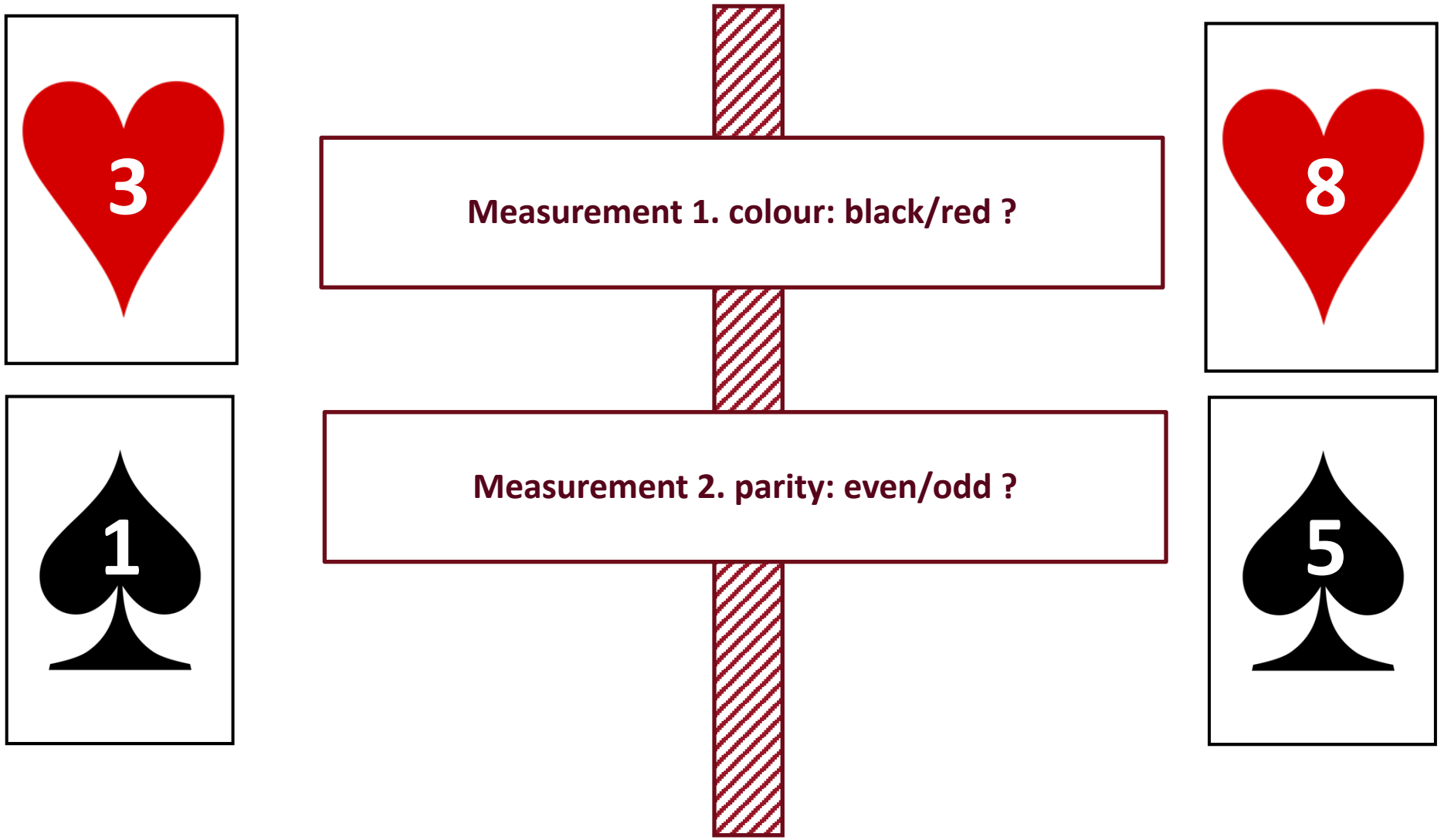
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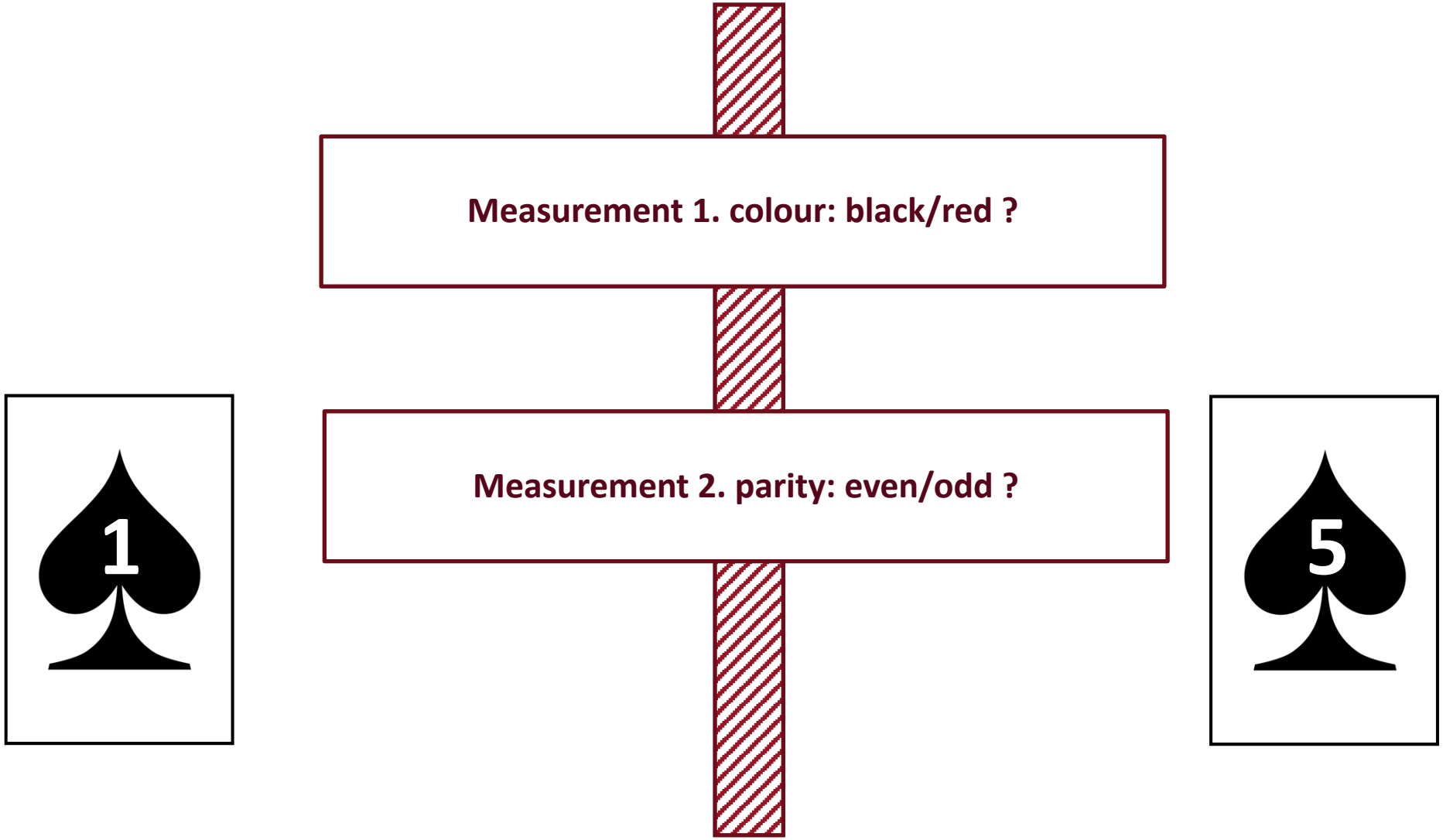
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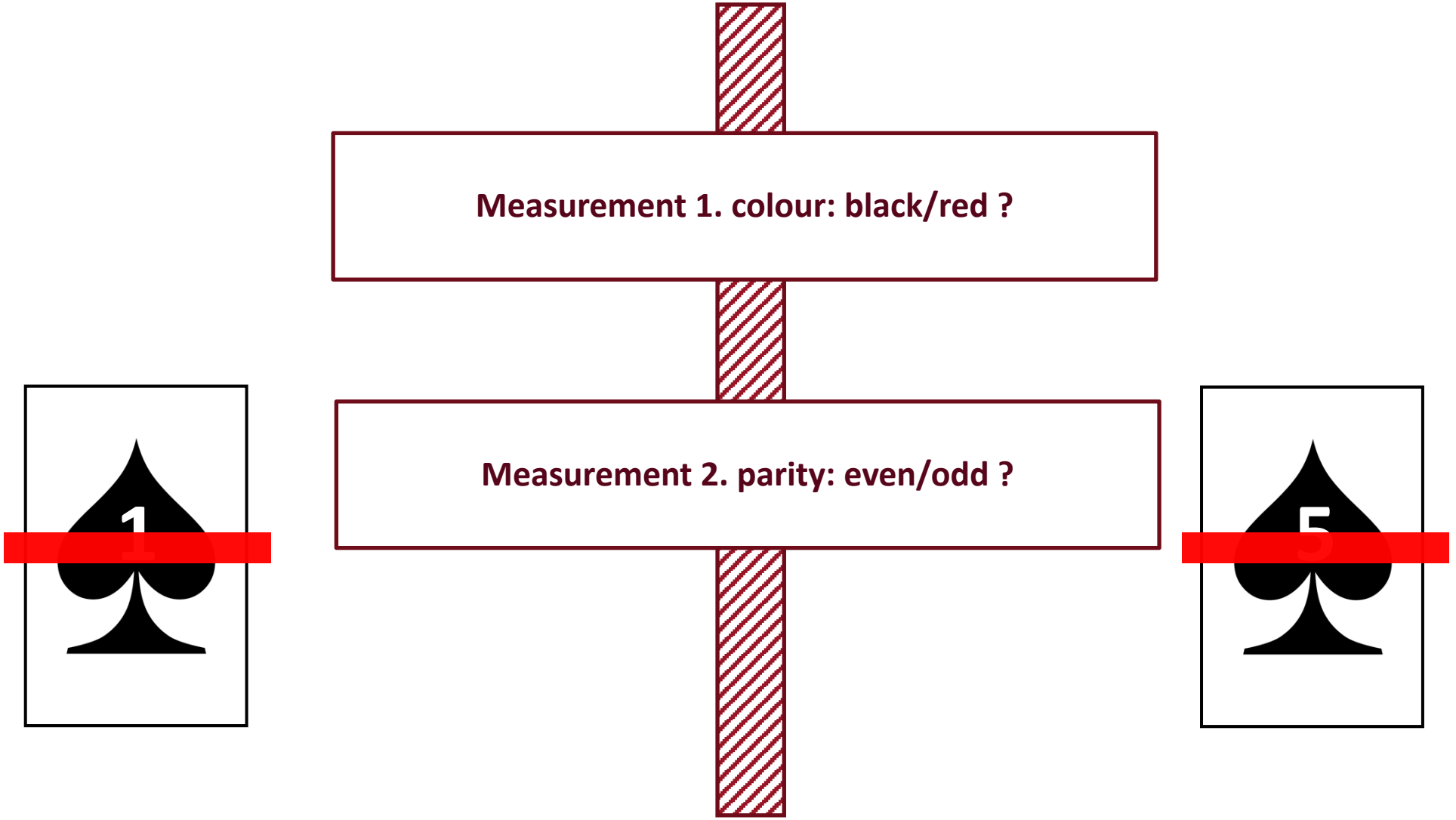
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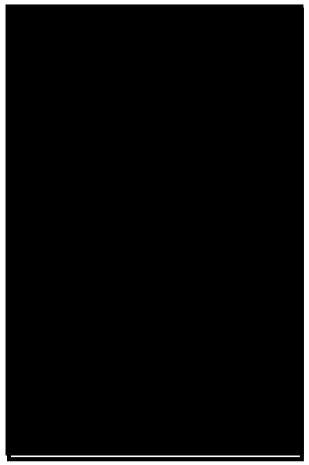


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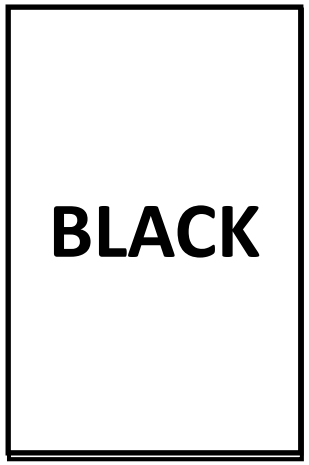
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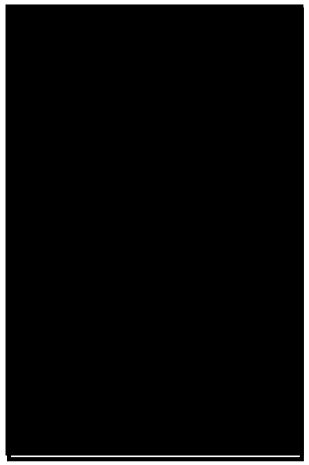
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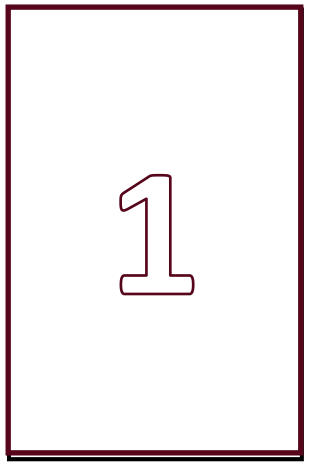
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Classical versus quantum properties

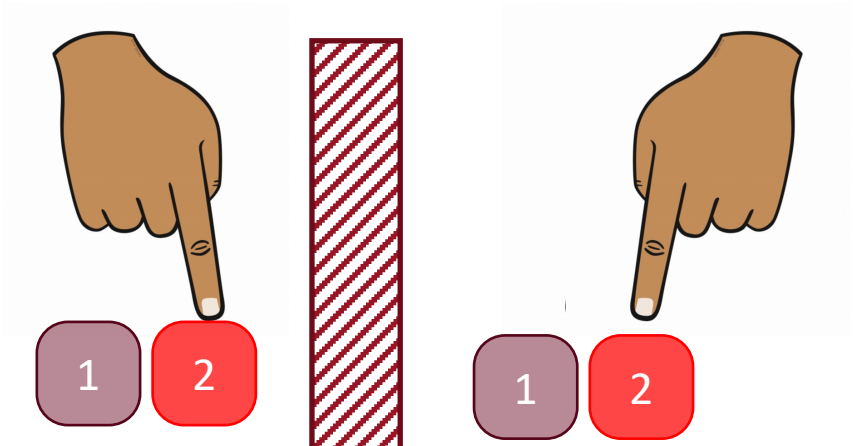
In quantum theory there exist measurements which do not commute. This means that the observables can't be diagonalised simultaneously in the same basis, which means they *cannot be simultaneously known.*



Measuring the spin in different and non-commuting directions $[\sigma_x, \sigma_z] \neq 0$.

Back to EPR

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The EPR state.

Density matrix

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle\,|\text{up}\rangle + |\text{down}\rangle\,|\text{down}\rangle) \quad \longrightarrow |\psi\rangle\langle\psi|$$

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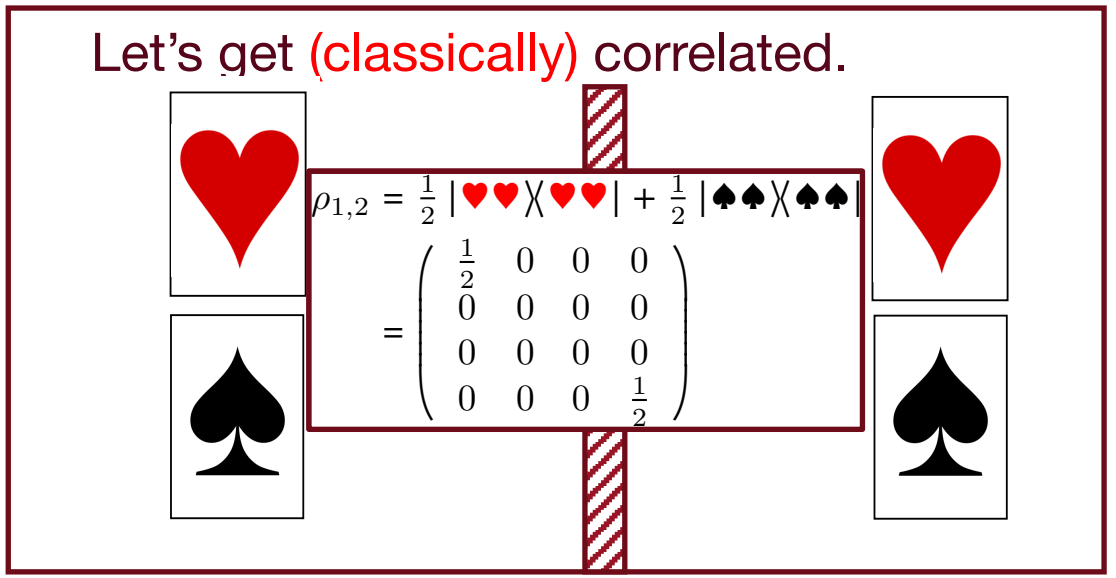
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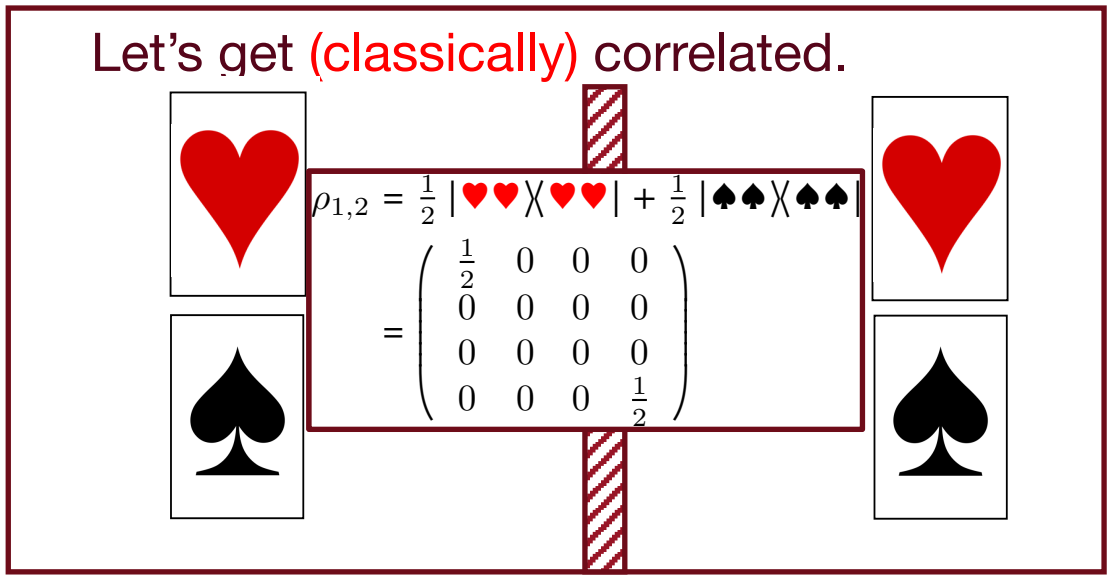
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Let's get (quantum) correlated.

Density matrix

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Entangled?

Two parties are said to be **entangled** if the quantum state of each party *cannot* be described independently of the pair. When this is the case, their density matrix cannot be written as a combination of separable states

Definition.

Entangled?

Two parties are said to be **entangled** if the quantum state of each party *cannot* be described independently of the pair. When this is the case, their density matrix cannot be written as a combination of separable states

$$\rho_{1,2} \neq \sum_i p_i \, \rho_1^i \otimes \rho_2^i$$

Entangled?

Two parties are said to be **entangled** if the quantum state of each party cannot be described independently of the pair. When this is the case, their density matrix cannot be written as a combination of separable states.

$$\rho_{1,2} = \tfrac{1}{4}\,|\heartsuit\heartsuit\rangle\langle\heartsuit\heartsuit| + \tfrac{1}{4}\,|\heartsuit\spadesuit\rangle\langle\heartsuit\spadesuit| + \tfrac{1}{4}\,|\spadesuit\heartsuit\rangle\langle\spadesuit\heartsuit| + \tfrac{1}{4}\,|\spadesuit\spadesuit\rangle\langle\spadesuit\spadesuit|$$

$$= \tfrac{1}{4}(|\heartsuit\rangle\langle\heartsuit| + |\spadesuit\rangle\langle\spadesuit|) \otimes (|\heartsuit\rangle\langle\heartsuit| + |\spadesuit\rangle\langle\spadesuit|)$$

$$= \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Separable

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Two parties are said to be **entangled** if the quantum state of each party cannot be described independently of the pair. When this is the case, their density matrix cannot be written as a combination of separable states.

$$\rho_{1,2} = \frac{1}{2}\,|\heartsuit\heartsuit\rangle\langle\heartsuit\heartsuit| + \frac{1}{2}\,|\spadesuit\spadesuit\rangle\langle\spadesuit\spadesuit|$$

$$= \frac{1}{2}(|\heartsuit\rangle\langle\heartsuit| \otimes |\heartsuit\rangle\langle\heartsuit|) + \frac{1}{2}(|\spadesuit\rangle\langle\spadesuit| \otimes |\spadesuit\rangle\langle\spadesuit|)$$

$$= \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Separable

Entangled.

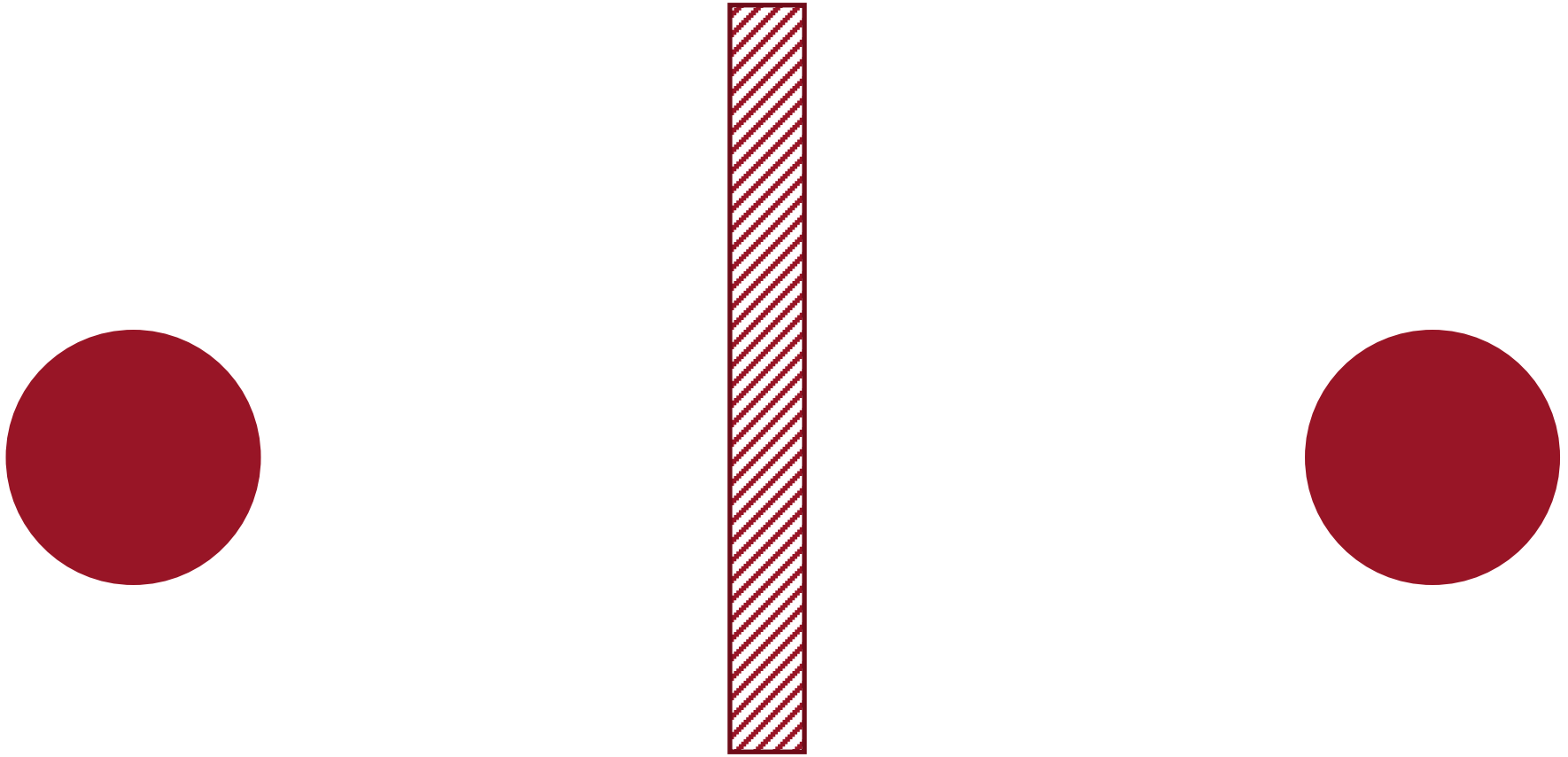
Two parties are said to be **entangled** if the quantum state of each party cannot be described independently of the pair. When this is the case, their density matrix cannot be written as a combination of separable states

$$\rho_{1,2} = |♥♥\rangle\langle♥♥| + |♥♥\rangle\langle♠♠| + |♠♠\rangle\langle♥♥| + |♠♠\rangle\langle♠♠|$$

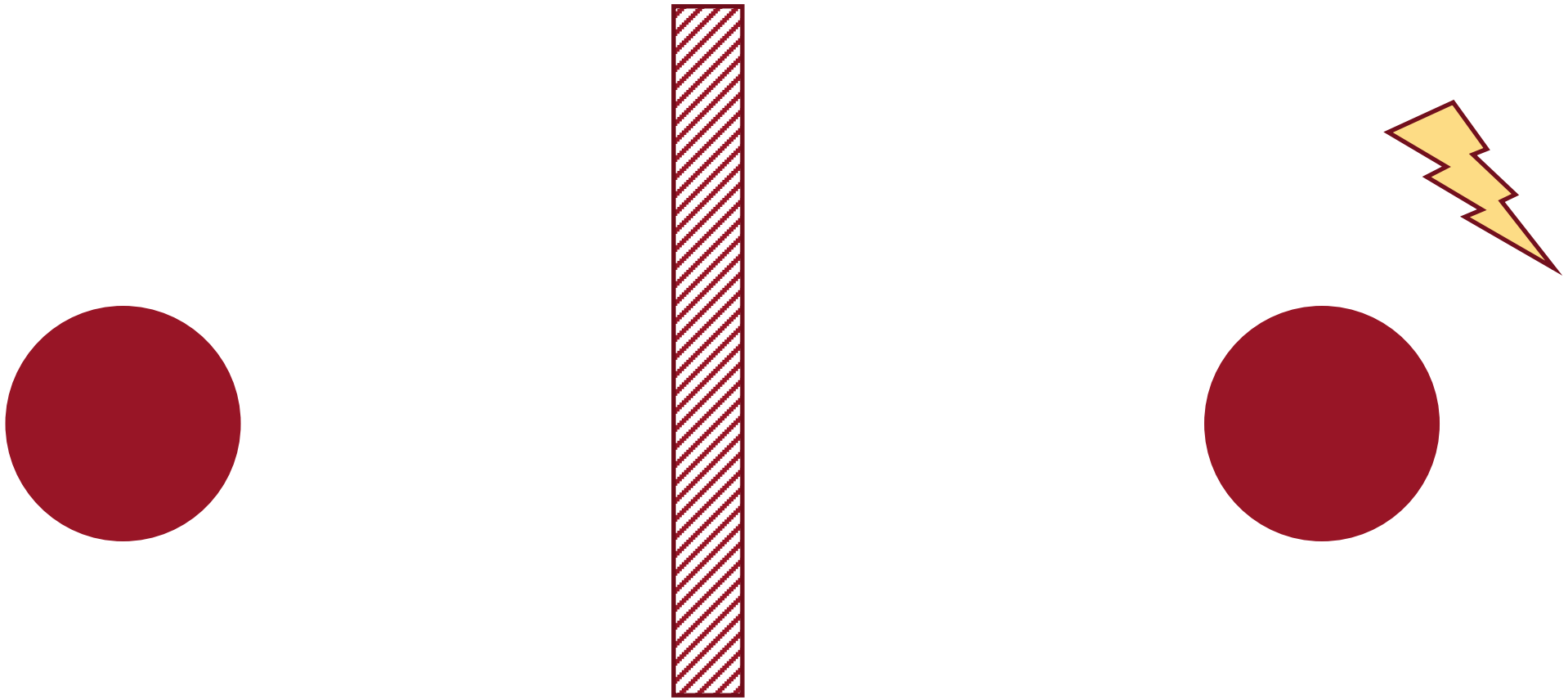
$$= \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Entangled

EPR thought experiment

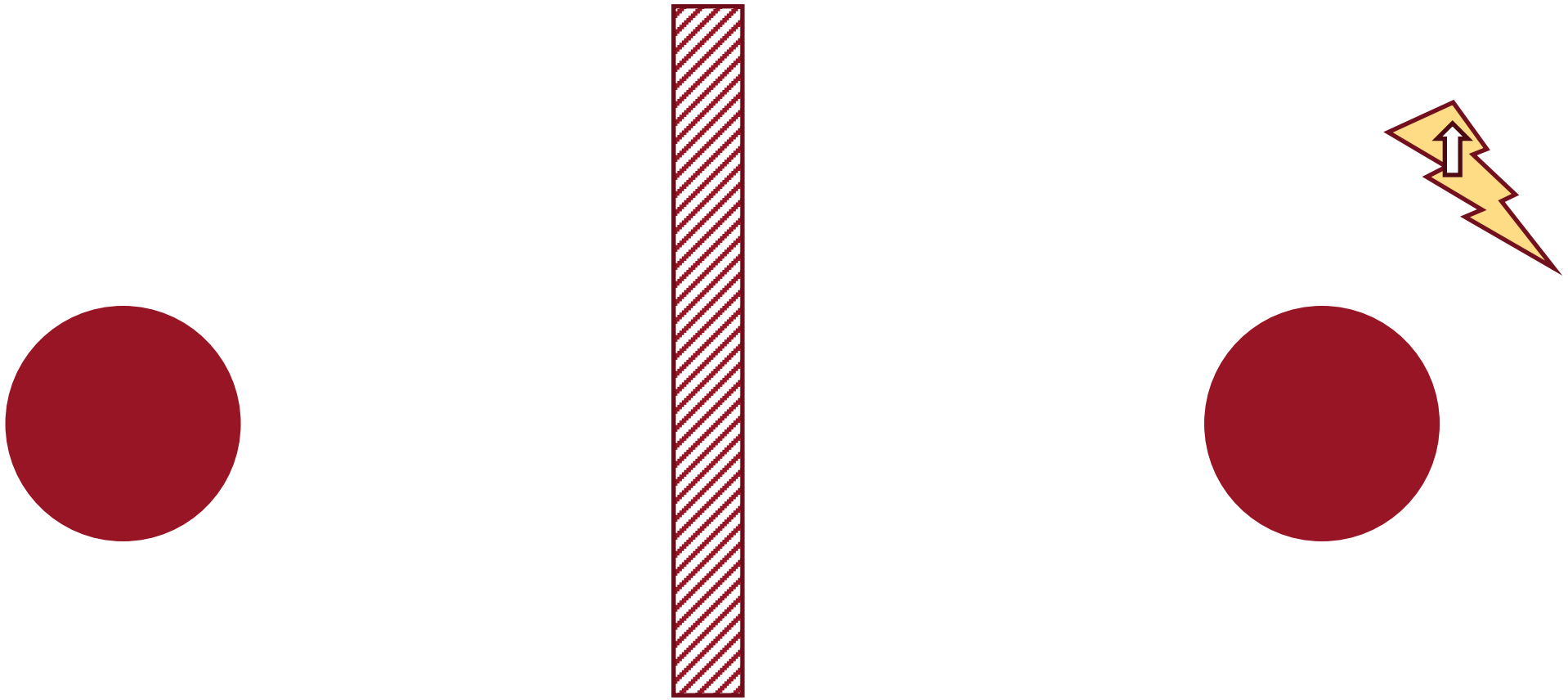


EPR thought experiment



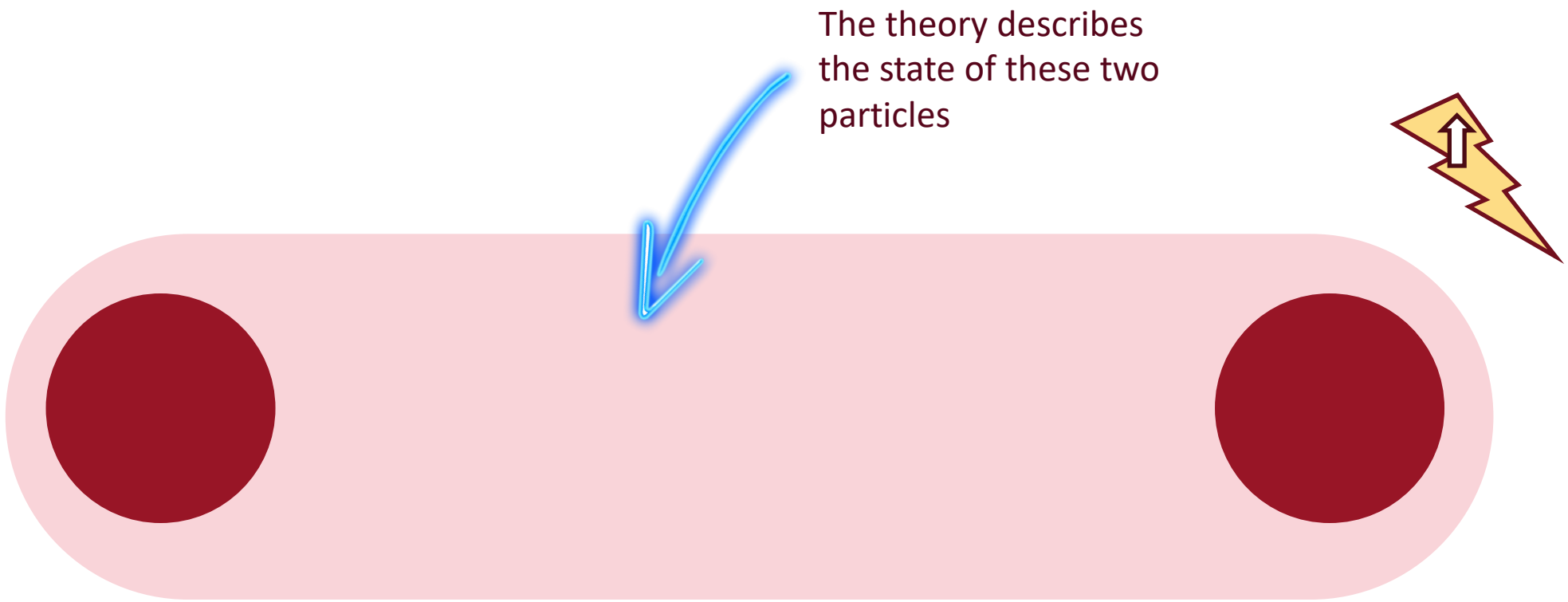
To avoid “spooky action at a distance” (i.e. to preserve assumption 2) EPR have to attribute an “element of physical reality” to the spin of particle 2, which predetermines the outcome of the measurement.

EPR thought experiment



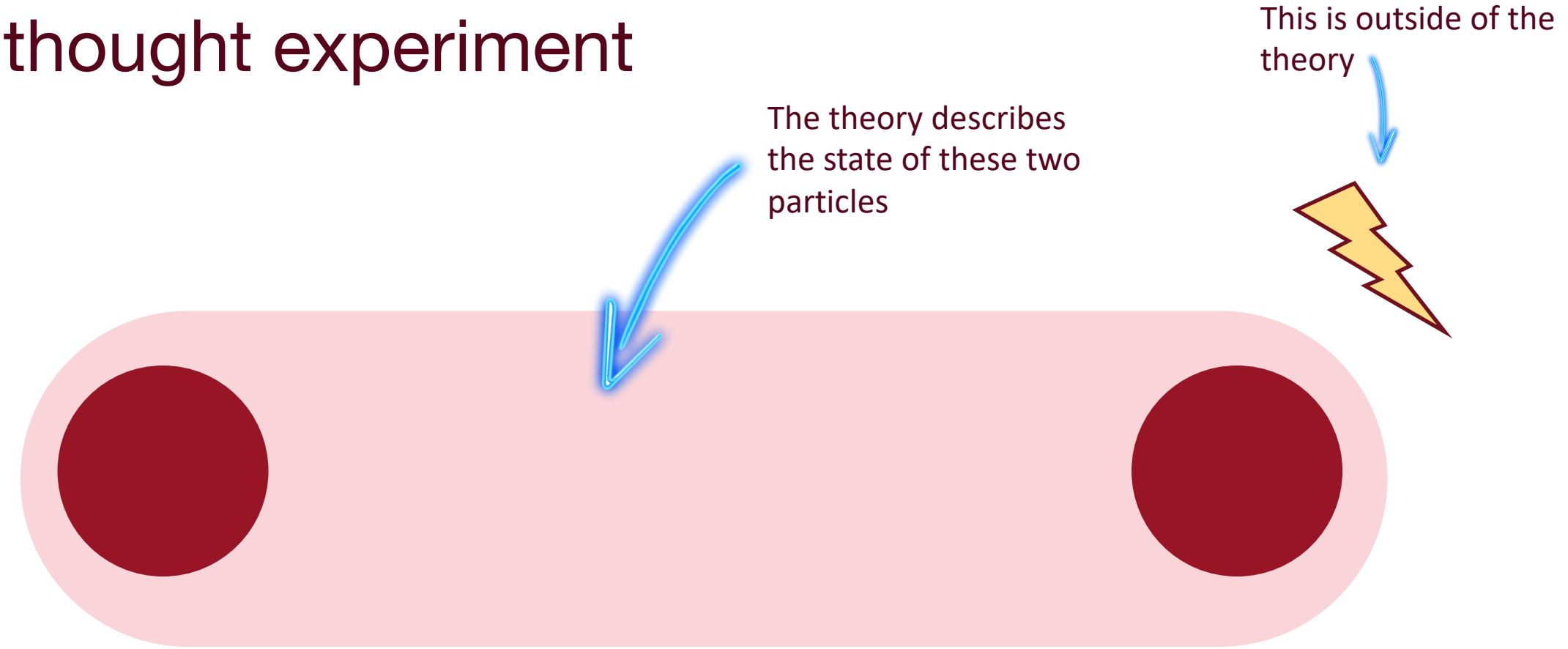
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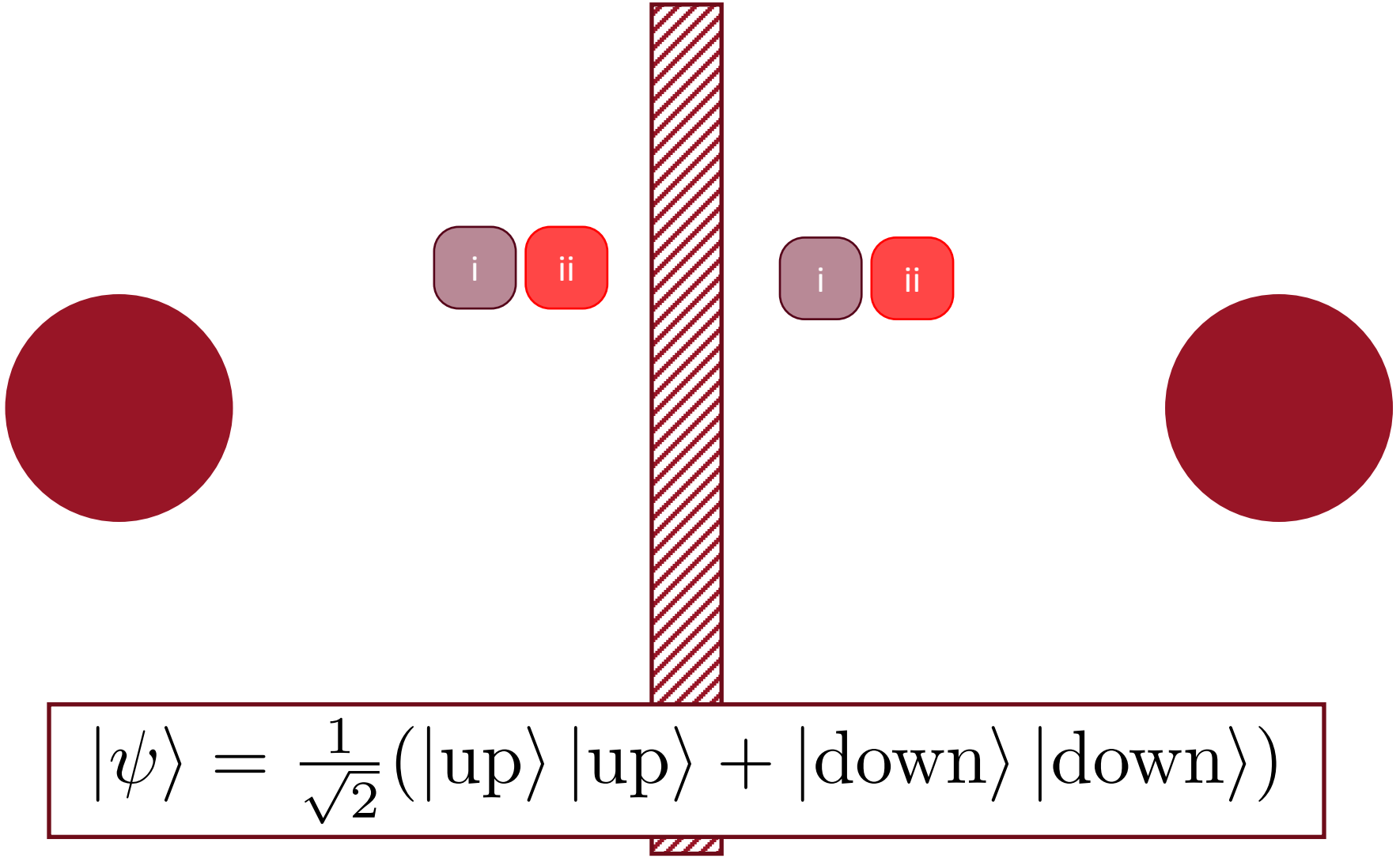
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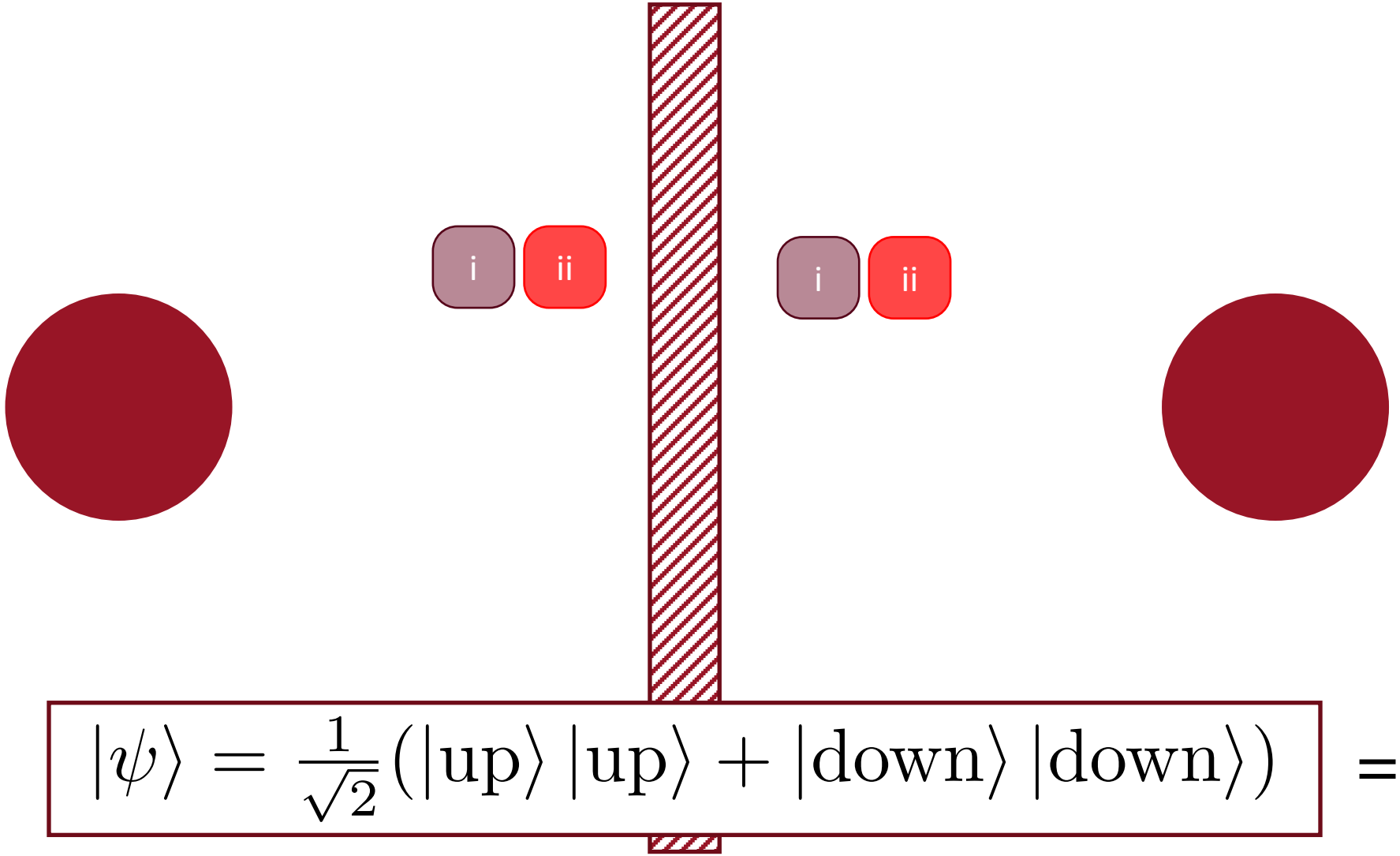
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EPR thought experiment



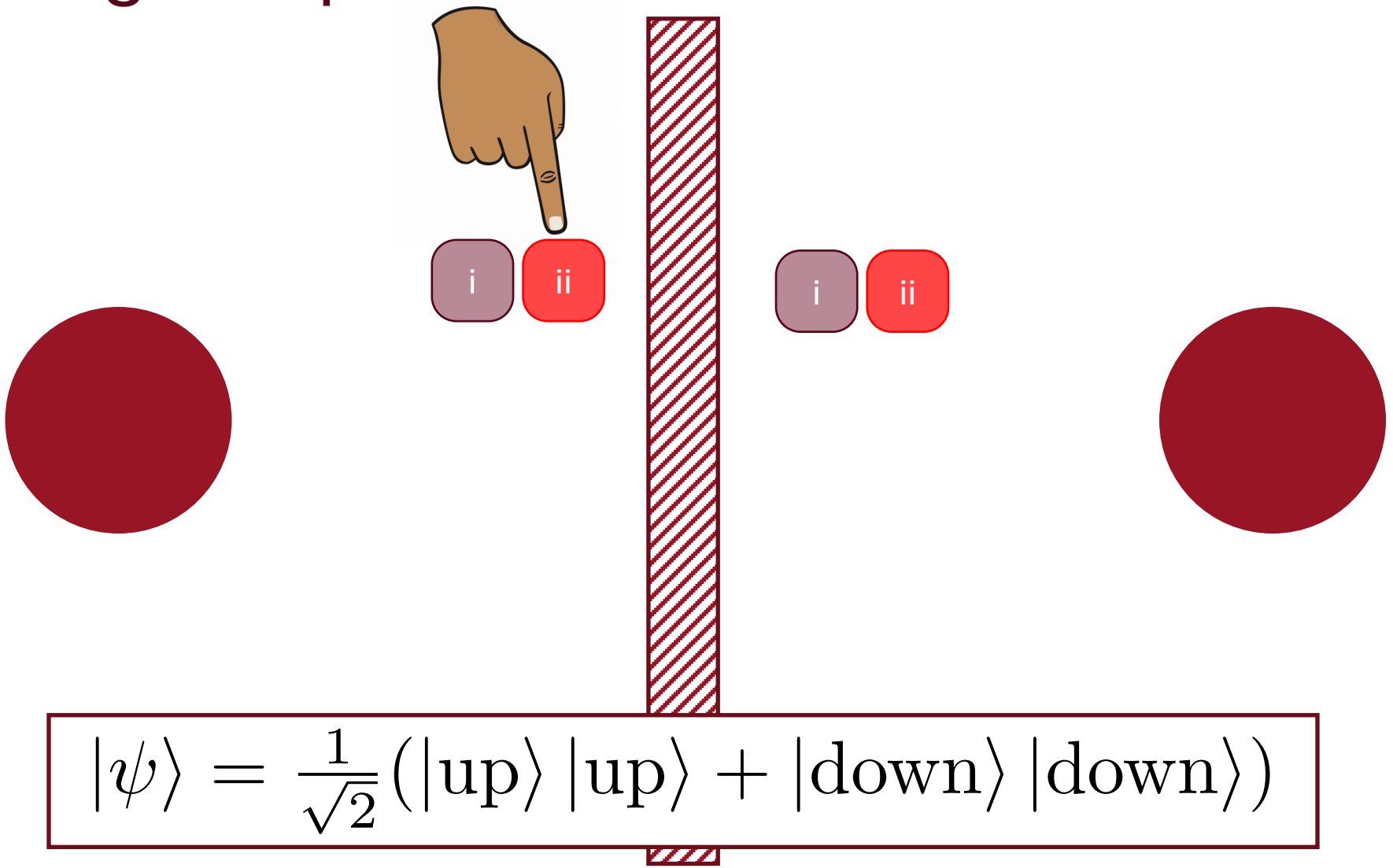
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle\,|\text{up}\rangle + |\text{down}\rangle\,|\text{down}\rangle)$$

EPR thought experiment



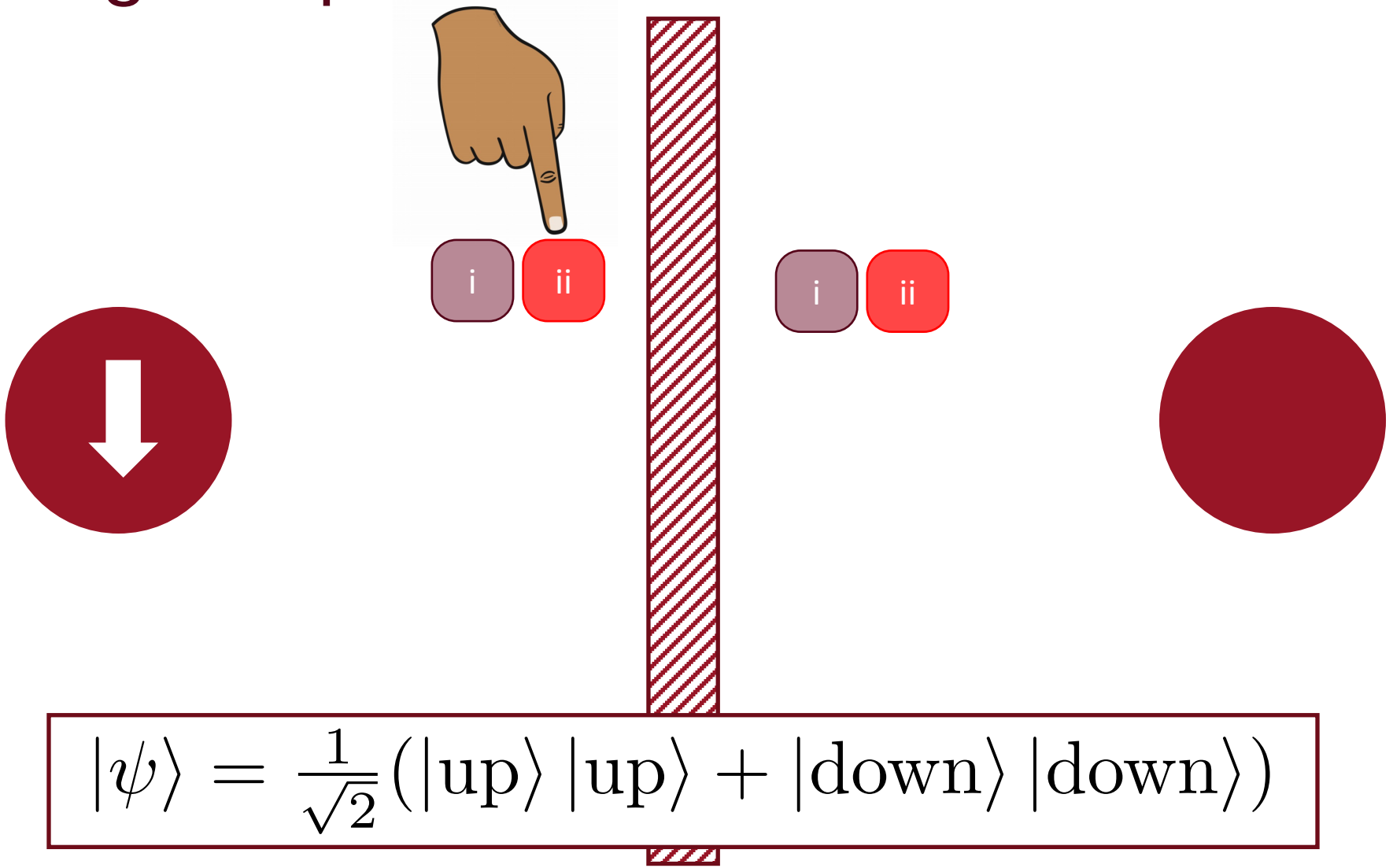
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle\,|\text{up}\rangle + |\text{down}\rangle\,|\text{down}\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

EPR thought experiment



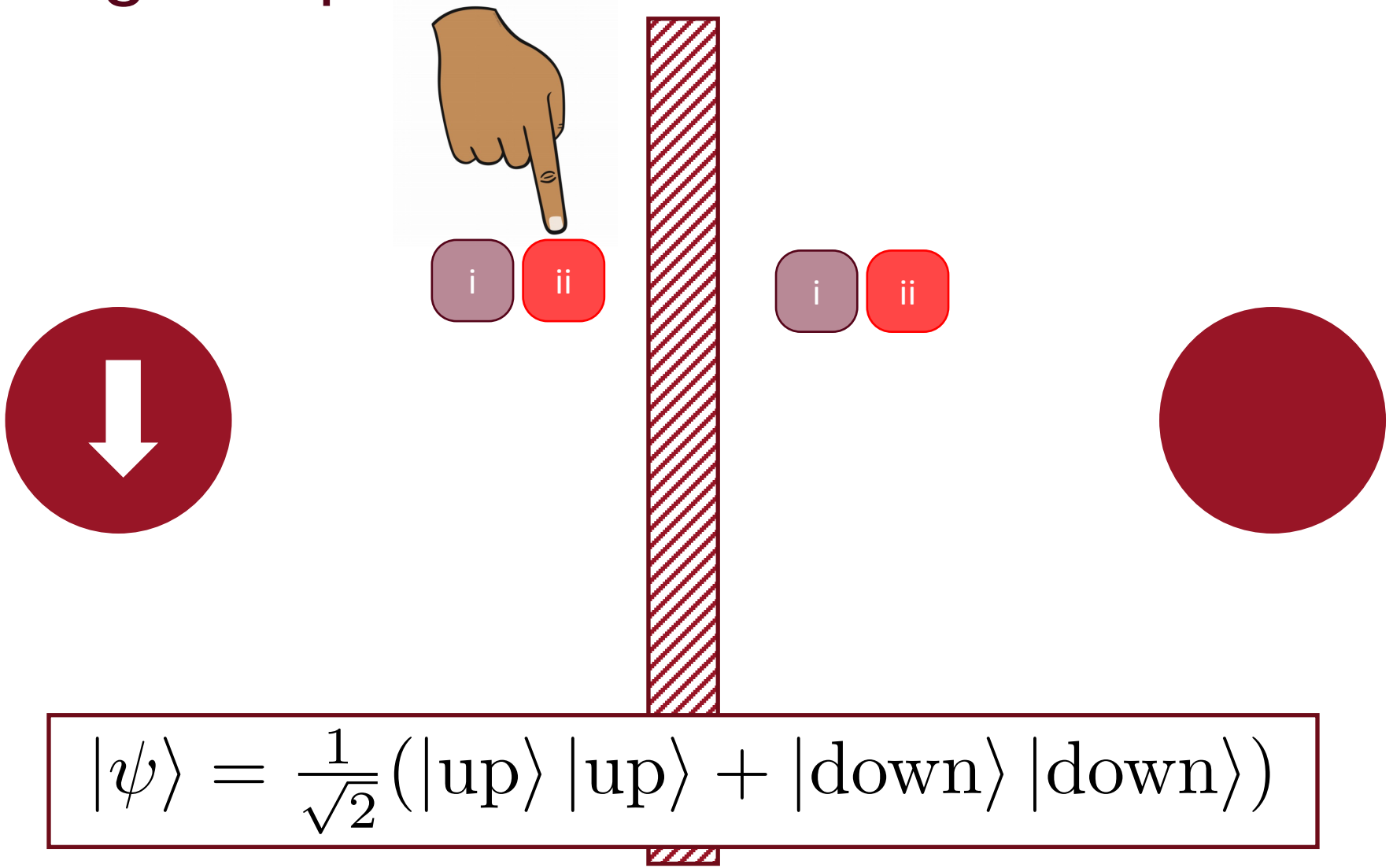
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EPR thought experiment



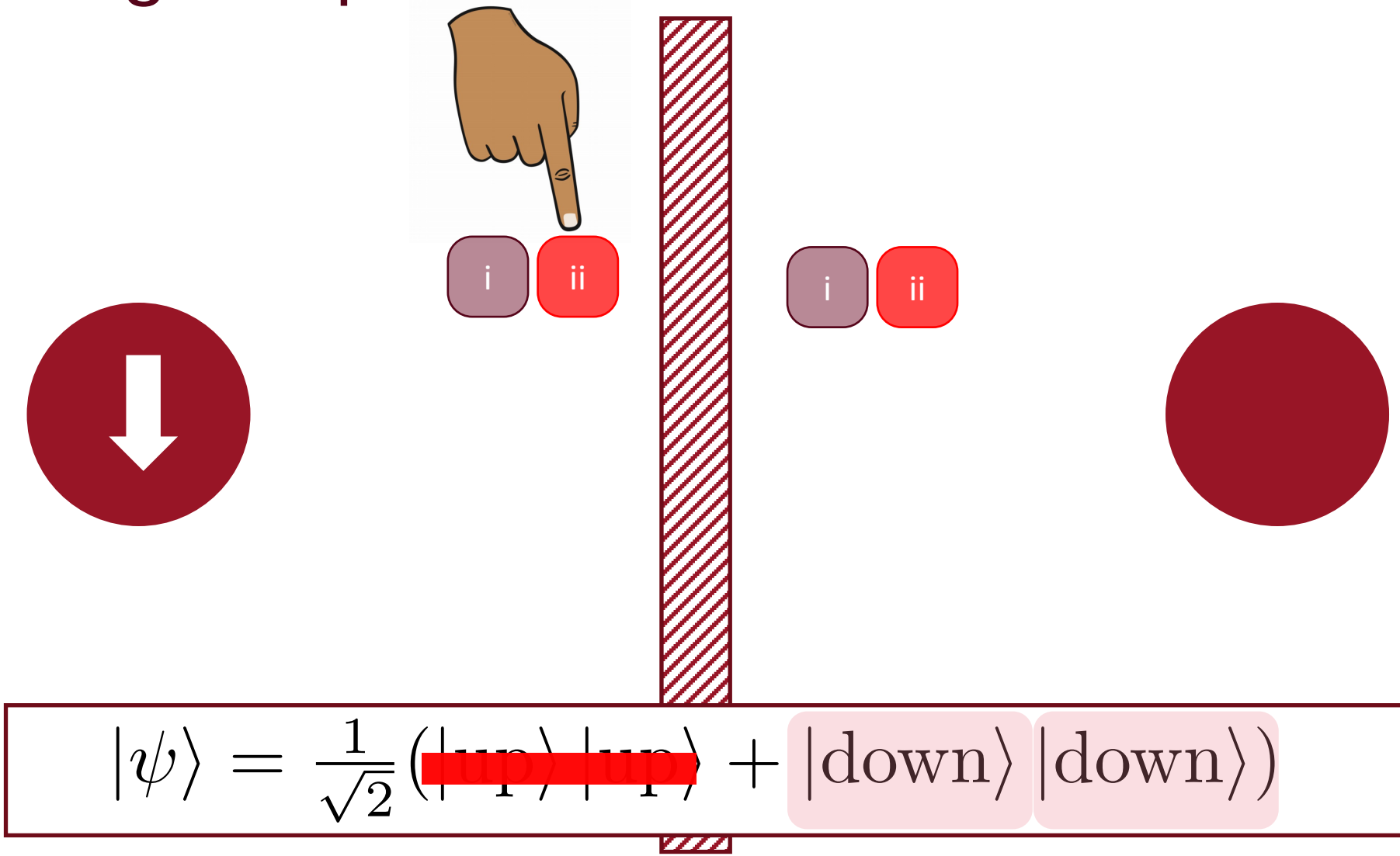
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EPR thought experiment



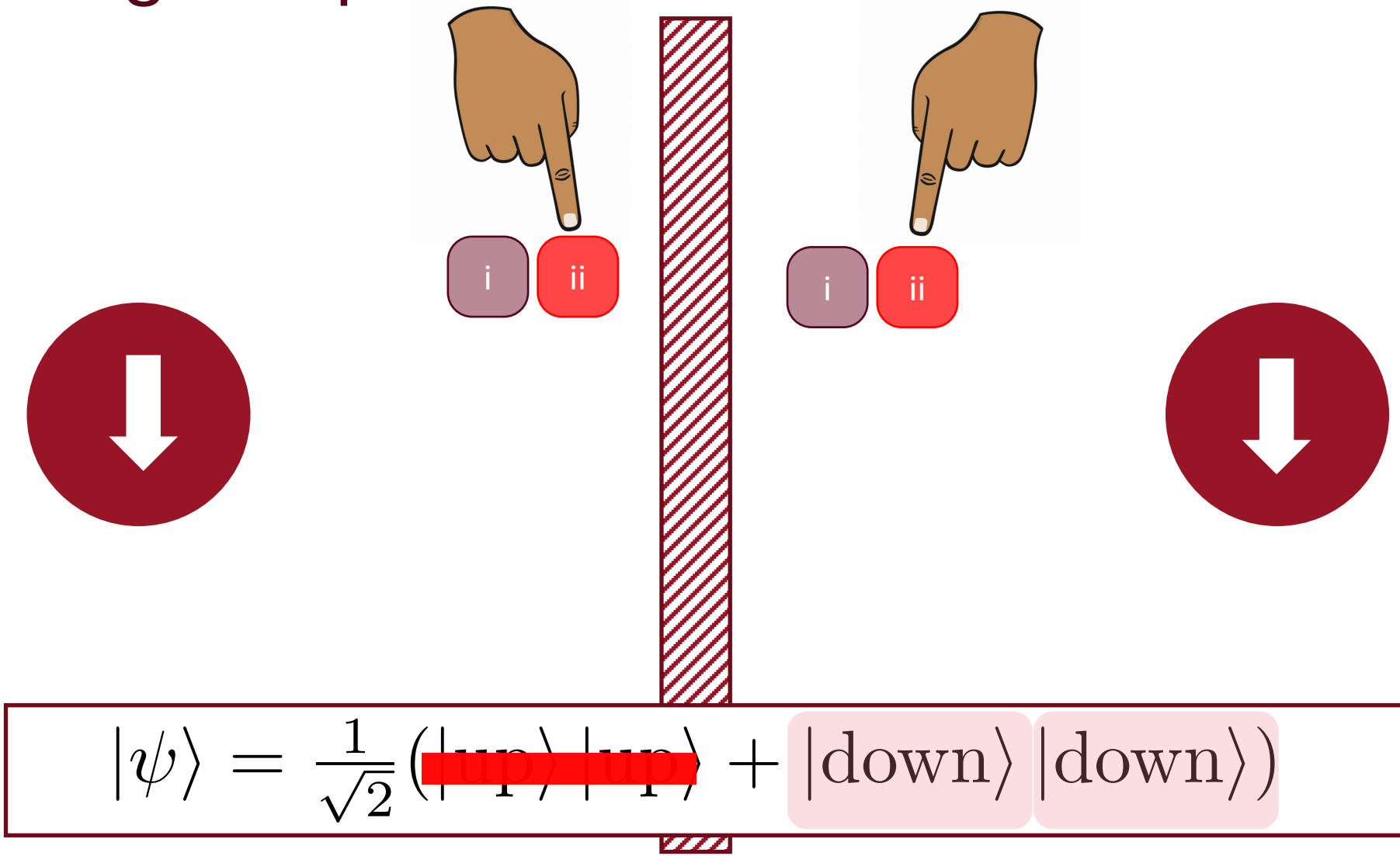
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EPR thought experiment



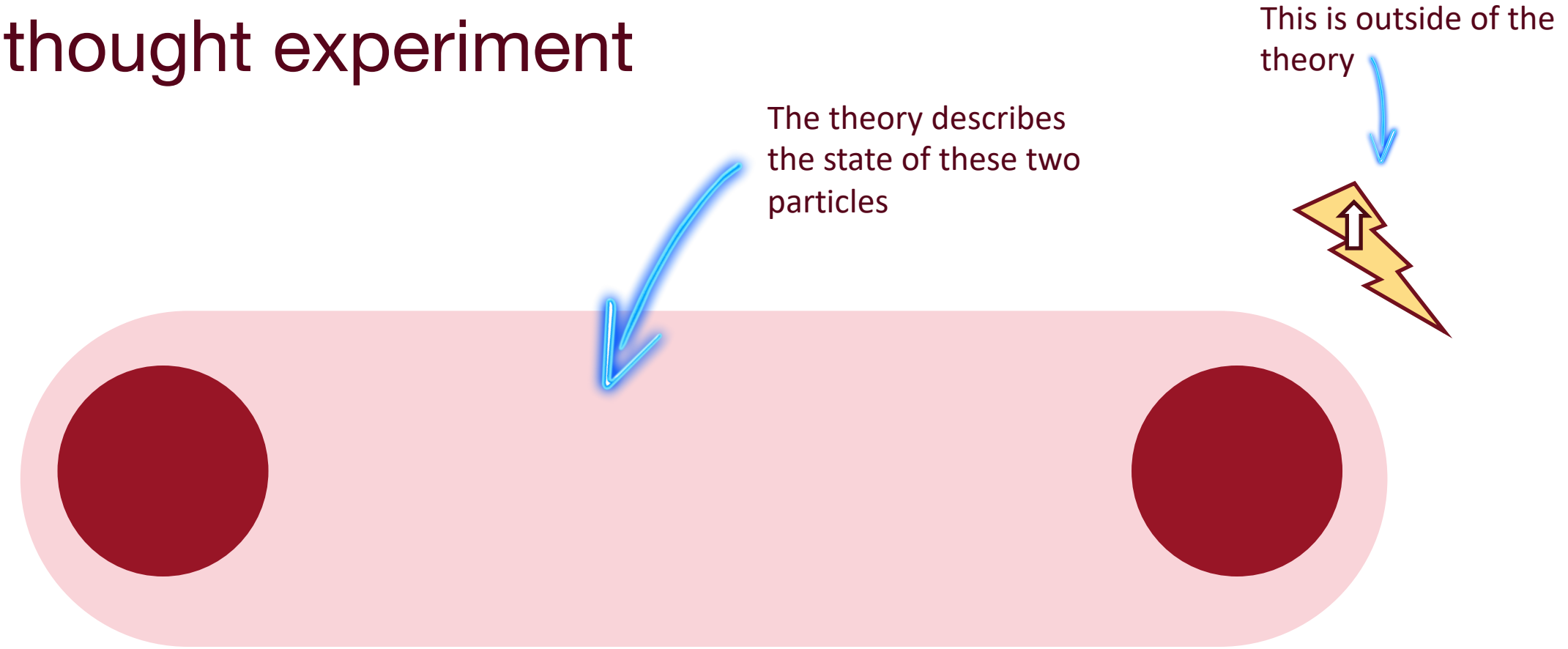
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EPR thought experiment



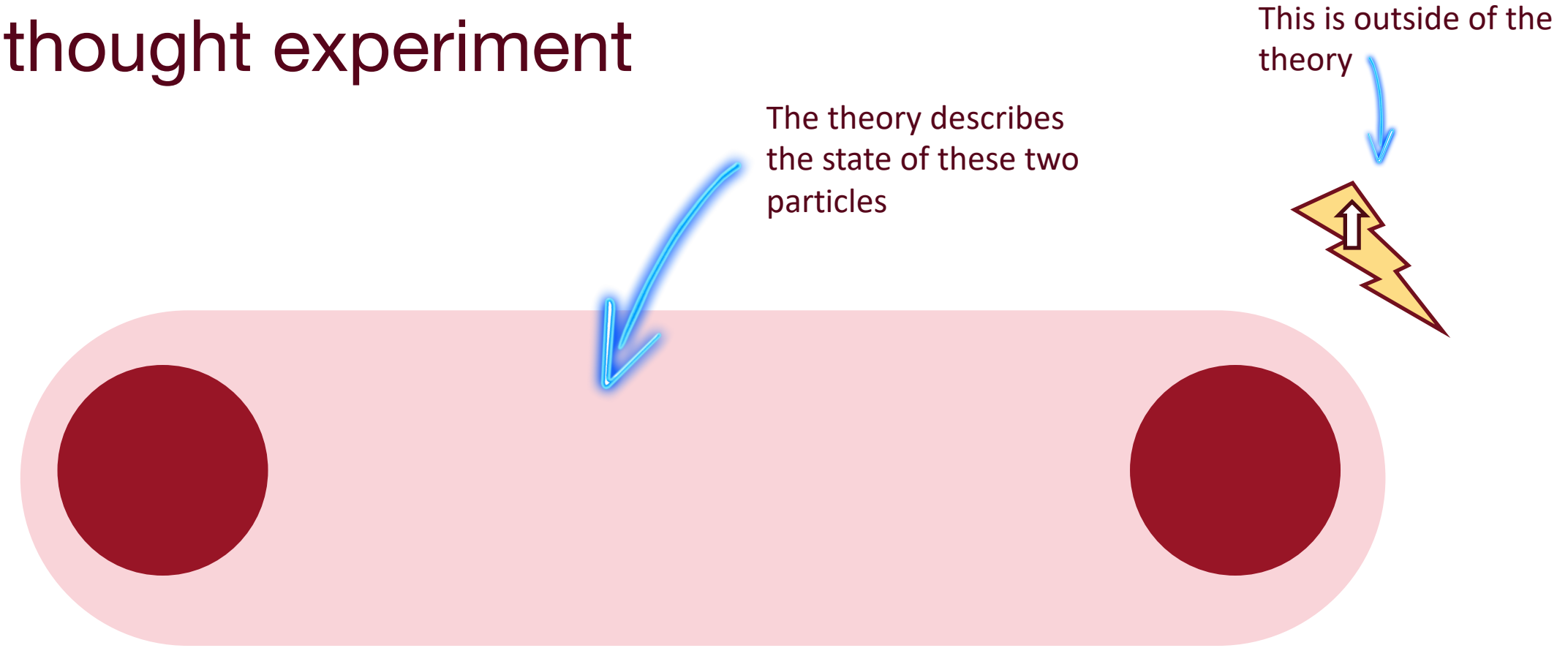
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{up}\rangle|\text{up}\rangle + |\text{down}\rangle|\text{down}\rangle)$$

EPR thought experiment



The quantum-mechanical formalism is incomplete!

EPR thought experiment



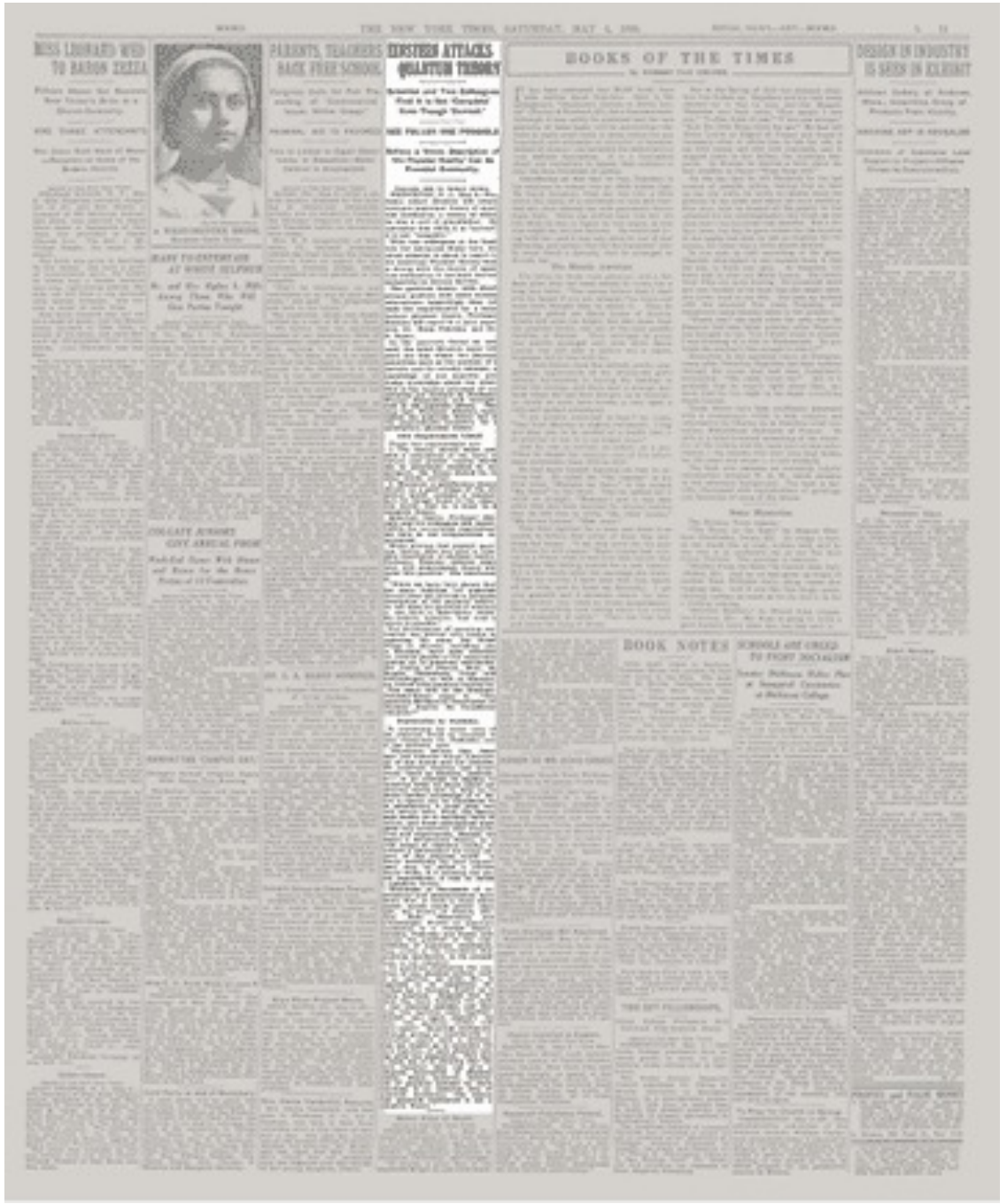
An extended theory, **deterministic hidden-variable theory,** is required if one were to take the EPR point of view.

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

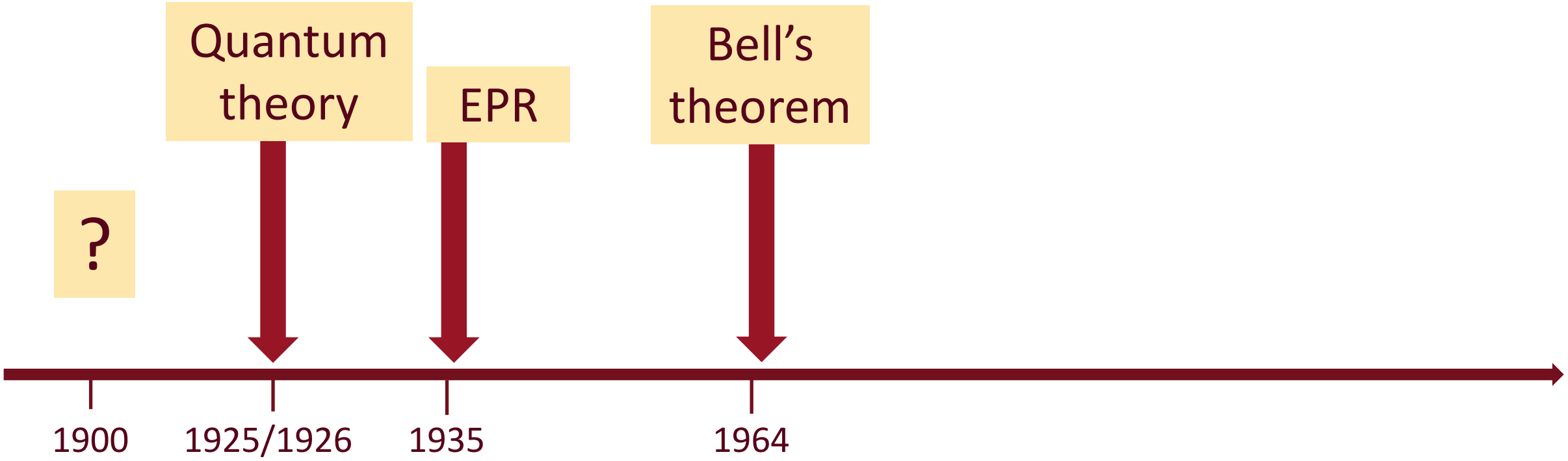
SEE FULLER ONE POSSIBLE

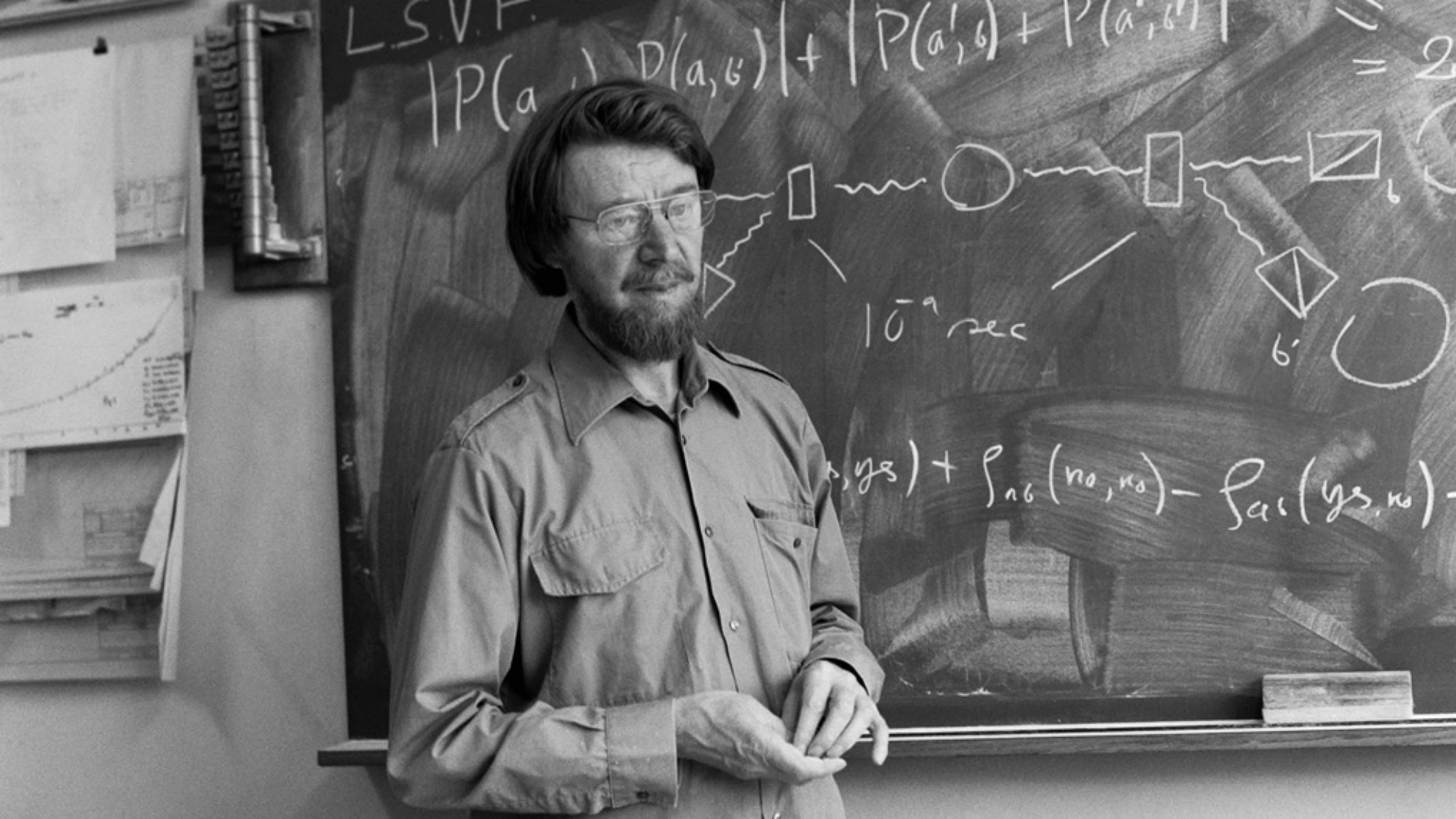
Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.



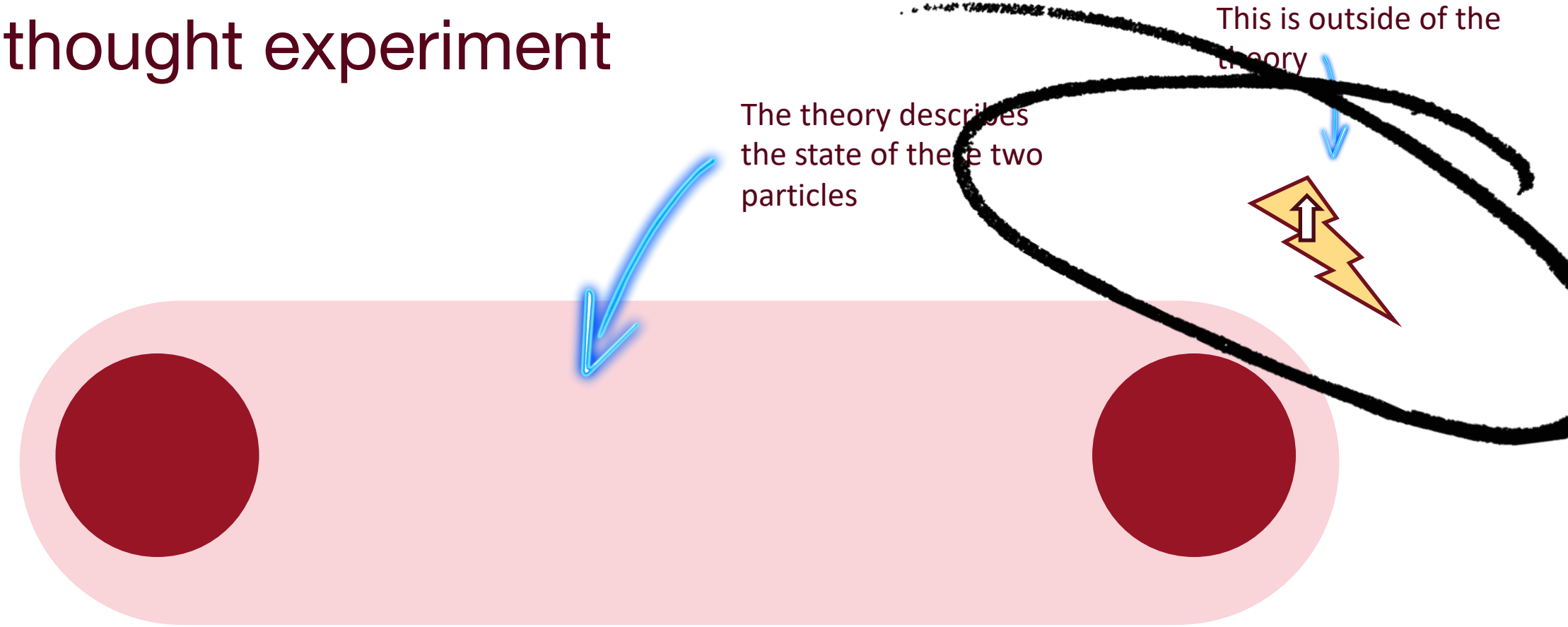
May 4, 1935 issue of *The New York Times*.

Quantum history condensed.




L.S.V.F.
10^{-9} sec

EPR thought experiment



Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

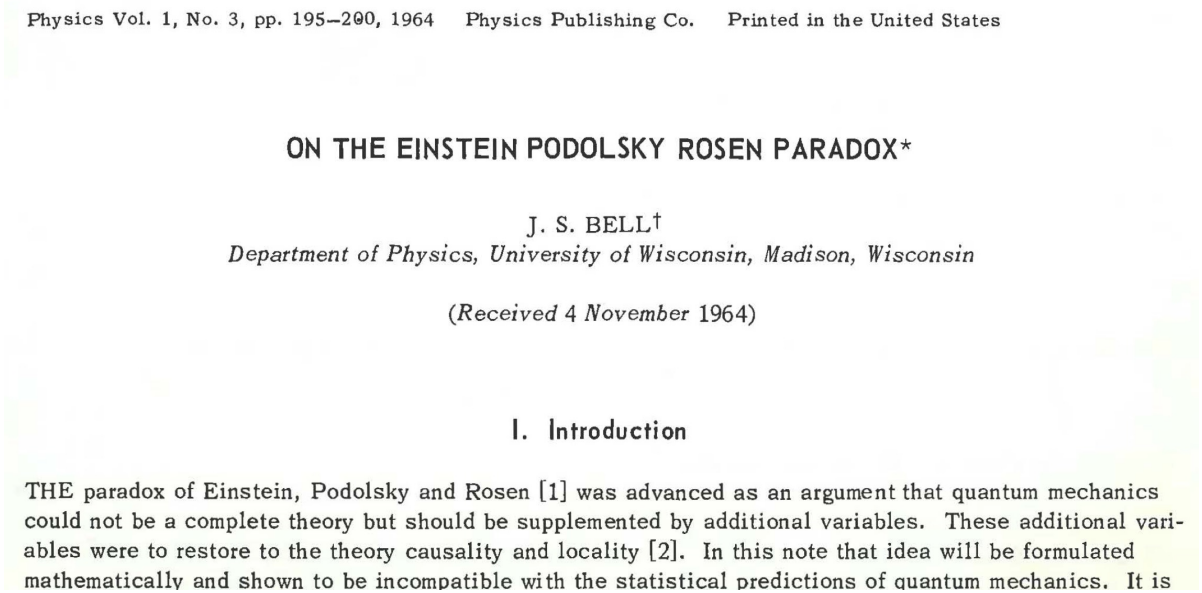
ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is



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John Bell was the first person to notice that some quantum correlations cannot be written with a local hidden variable model.

But where should one look for these correlations?

What is the experiment?

The CHSH ‘game’ – the test for Bell’s theorem.

J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

$$CHSH = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

PROPOSED EXPERIMENT TO TEST LOCAL HIDDEN-VARIABLE THEORIES*

John F. Clauser†
Department of Physics, Columbia University, New York, New York 10027

and

Michael A. Horne
Department of Physics, Boston University, Boston, Massachusetts 02215

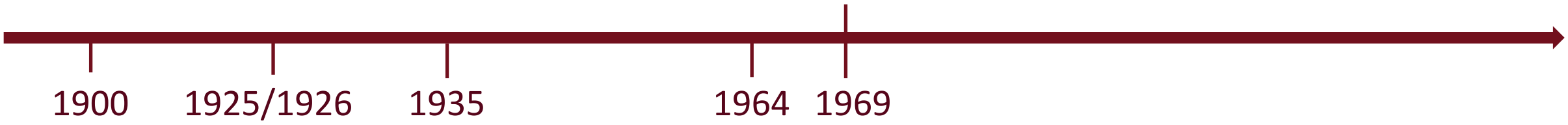
and

Abner Shimony
Departments of Philosophy and Physics, Boston University, Boston, Massachusetts 02215

and

Richard A. Holt
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A theorem of Bell, proving that certain predictions of quantum mechanics are inconsistent with the entire family of local hidden-variable theories, is generalized so as to apply to realizable experiments. A proposed extension of the experiment of Kocher and Commins, on the polarization correlation of a pair of optical photons, will provide a decisive test between quantum mechanics and local hidden-variable theories.



The CHSH ‘game’ – the test for Bell’s theorem.

If you side with EPR, then your experimental results would obey Bell’s inequality (a type of equation).

If you side with Bohr, then your experimental results would not obey this equation.

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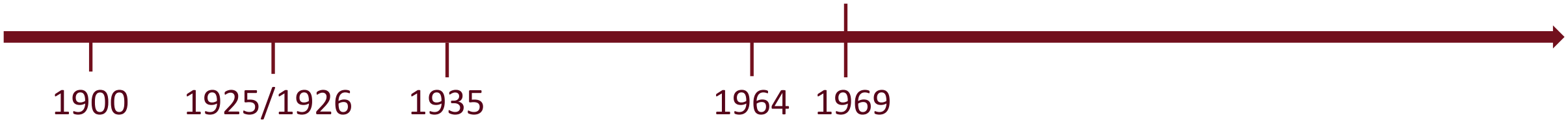
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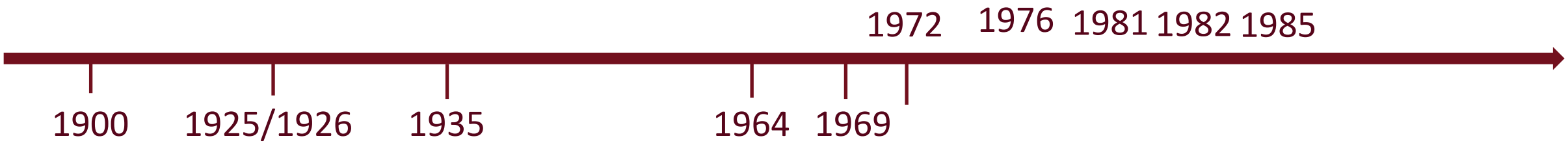
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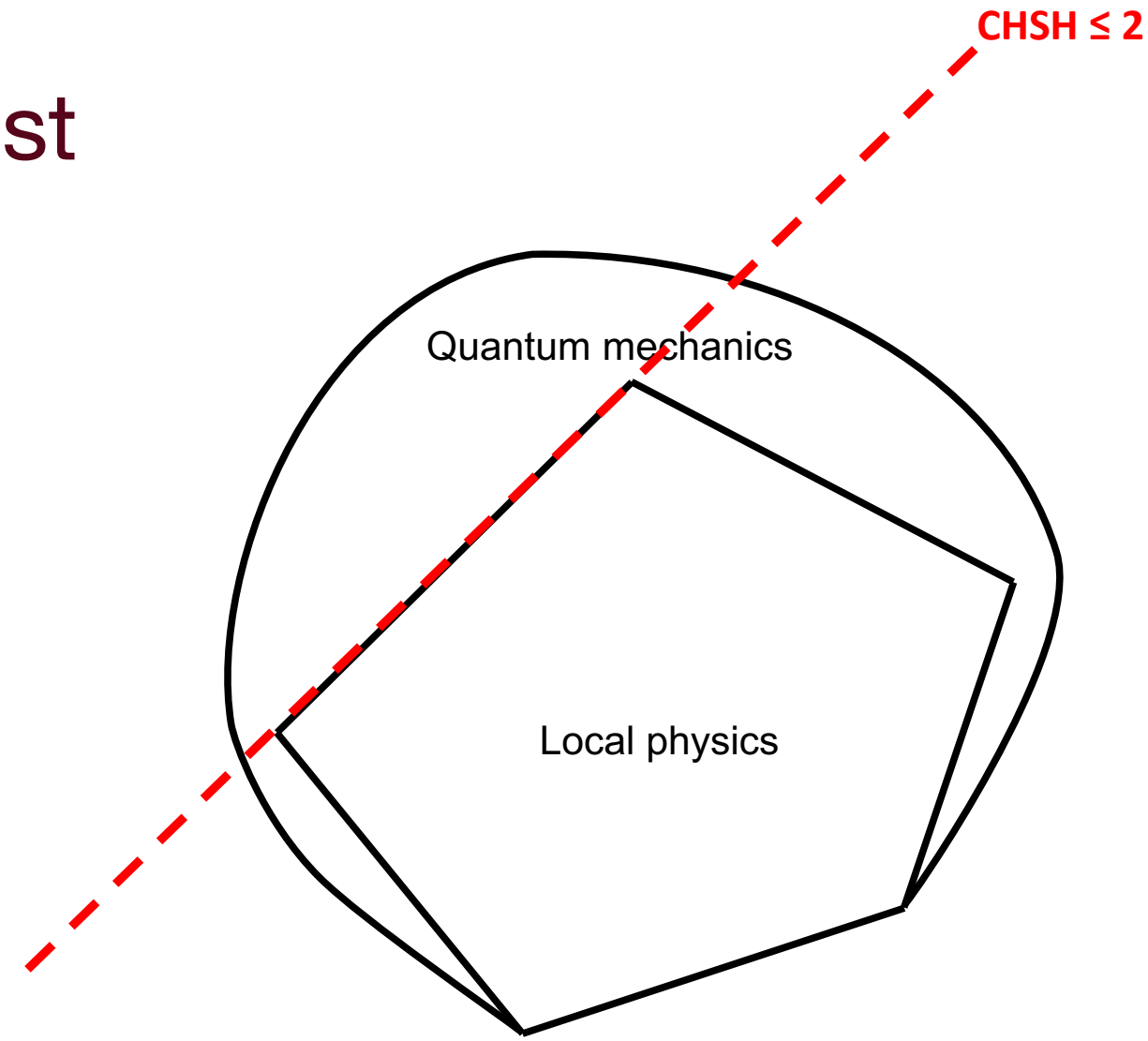
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The CHSH ‘game’ – the test for Bell’s theorem.



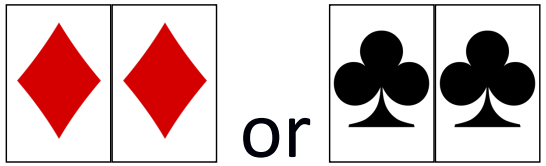
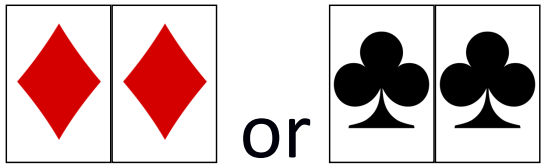
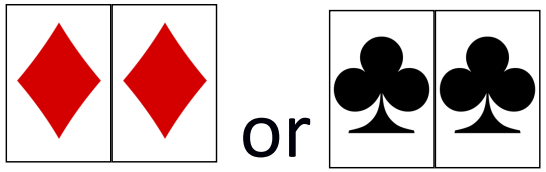
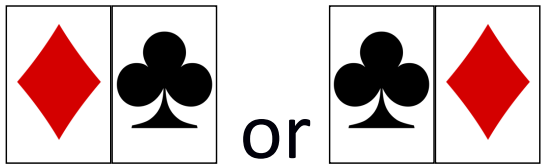
Bell test



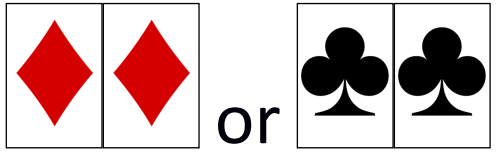
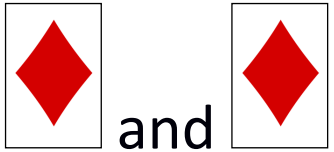
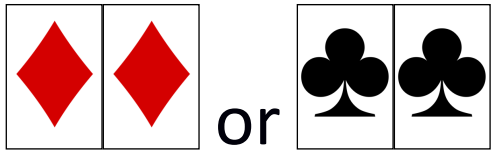
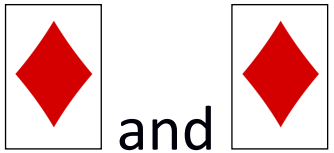
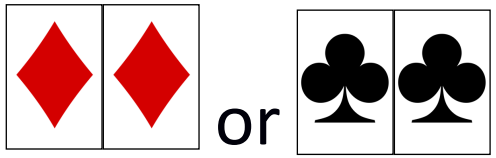
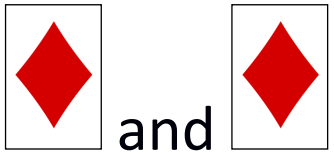
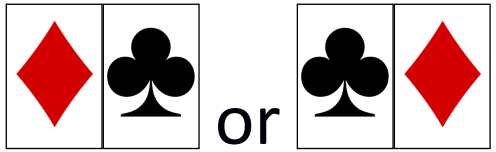
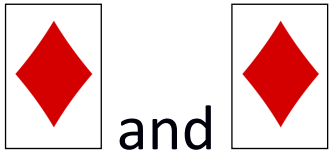
The Nobel Prize in Physics 2022 was awarded to Alain Aspect, John F. Clauser and Anton Zeilinger “for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science”.



Understanding the CHSH game

Alice's measurement	Bob's measurement	Winning reply
i	i	
i	ii	
ii	i	
ii	ii	

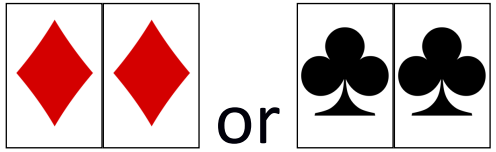
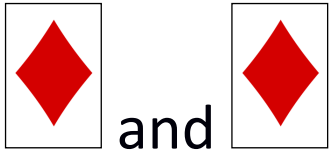
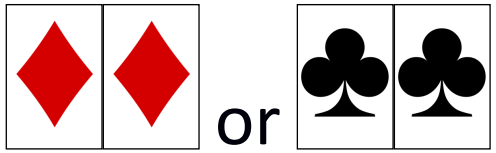
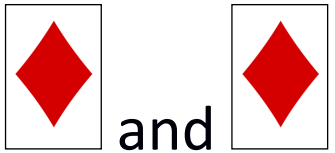
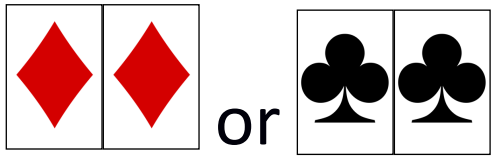
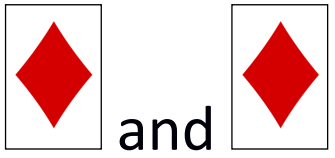
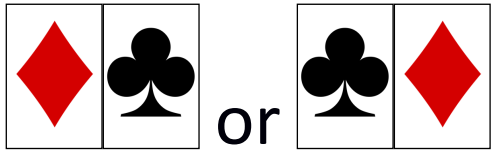
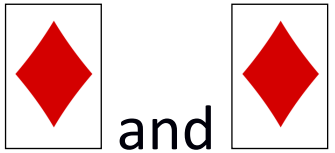
Understanding the CHSH game: strategy 1

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i		
i	ii		
ii	i		
ii	ii		

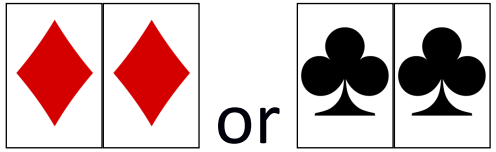
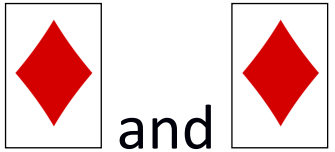
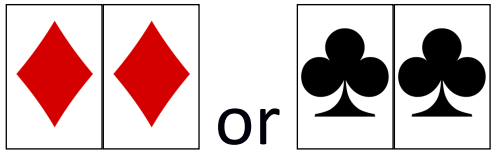
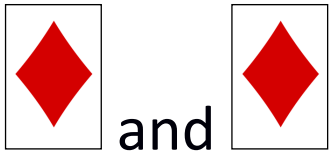
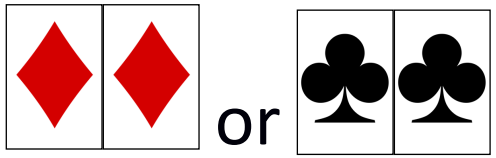
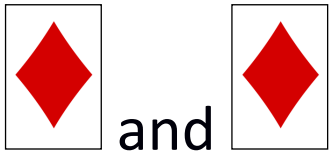
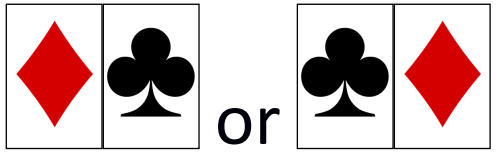
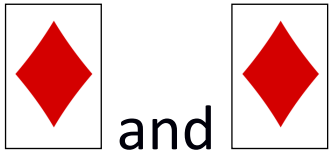
Understanding the CHSH game: strategy 1

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i	♦♦ or ♣♣	♦ and ♦
i	ii	♦♦ or ♣♣	♦ and ♦
ii	i	♦♦ or ♣♣	♦ and ♦
ii	ii	♦♣ or ♣♦	♦ and ♦

Understanding the CHSH game: strategy 1

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i		
i	ii		
ii	i		
ii	ii		

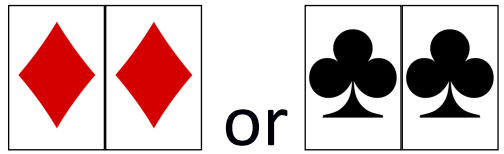
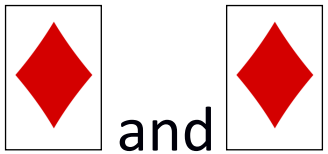
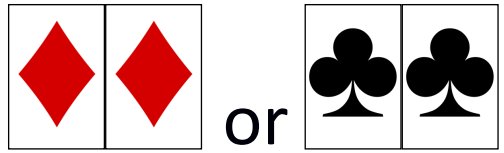
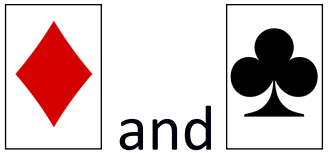
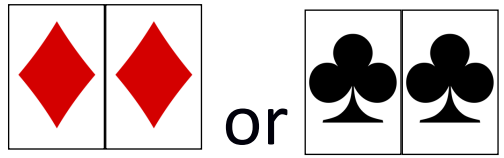
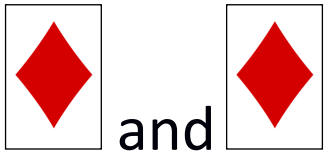
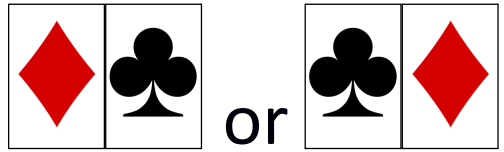
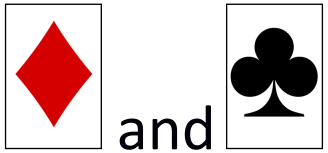
Understanding the CHSH game: strategy 1

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i		
i	ii		
ii	i		
ii	ii		

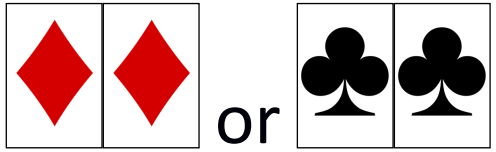
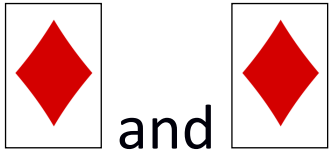
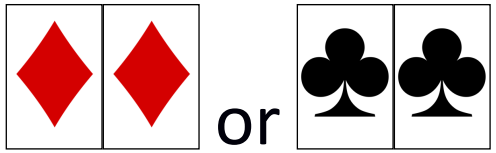
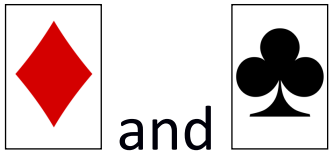
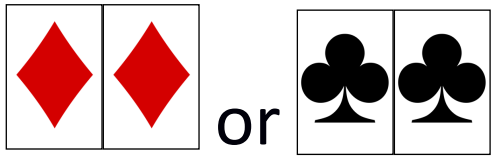
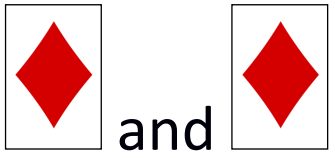
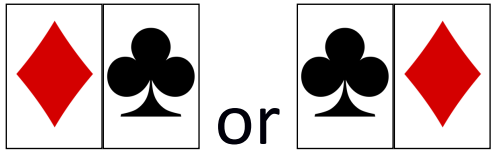
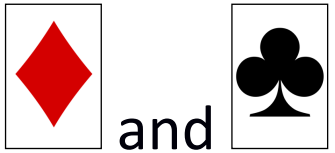
Understanding the CHSH game: strategy 1

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i	♦♦ or ♣♣	♦ and ♦ ✓
i	ii	♦♦ or ♣♣	♦ and ♦ ✓
ii	i	♦♦ or ♣♣	♦ and ♦ ✓
ii	ii	♦♣ or ♣♦	♦ and ♦ ✗

Understanding the CHSH game: strategy 2

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i		
i	ii		
ii	i		
ii	ii		

Understanding the CHSH game: strategy 2

Alice's measurement	Bob's measurement	Winning reply	Strategy
i	i		
i	ii		
ii	i		
ii	ii		

Classical: 75%

Quantum: 85%

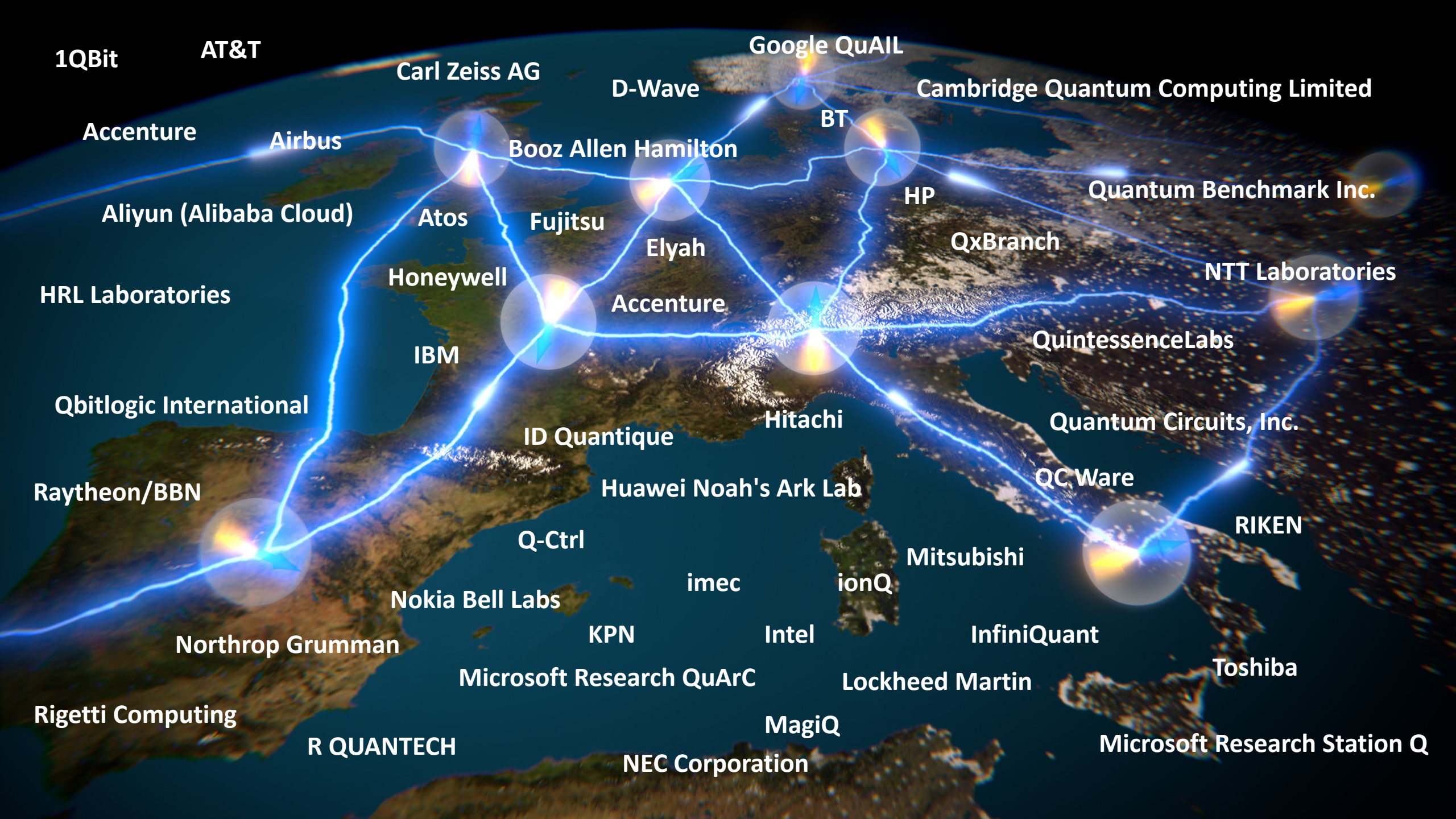
Classical: 75%

Quantum: 85%

There is no pre-arranged strategy, no probabilistic strategy that one can devise (not even using all the classical supercomputing resources available in the universe!) that can beat the classical bound of 75% success.

What can we do with a quantum advantage?

Quantum communication protocols, quantum teleportation, entanglement swapping, quantum key distribution, quantum computing…


1QBit
AT&T
Carl Zeiss AG
Google QuAIL
D-Wave
Cambridge Quantum Computing Limited
Accenture
Airbus
BT
Booz Allen Hamilton
Aliyun (Alibaba Cloud)
Atos
Fujitsu
HP
Quantum Benchmark Inc.
Elyah
QxBranch
Honeywell
NTT Laboratories
HRL Laboratories
Accenture
IBM
QuintessenceLabs
Qbitlogic International
Hitachi
Quantum Circuits, Inc.
ID Quantique
QC Ware
Raytheon/BBN
Huawei Noah's Ark Lab
RIKEN
Q-Ctrl
Mitsubishi
imec
ionQ
Nokia Bell Labs
Northrop Grumman
KPN
Intel
InfiniQuant
Toshiba
Microsoft Research QuArC
Lockheed Martin
Rigetti Computing
MagiQ
R QUANTECH
Microsoft Research Station Q
NEC Corporation

Entanglement wrap up.

- Entanglement is a property of two or more systems. It is present when the description of the total state *doesn't* admit a particular decomposition (factorisation*).

- Entangled states can produce *non-local* correlations. When they do, it may be possible to detect entanglement.

- All pure states violate a Bell inequality but not all mixed states violate a Bell inequality.

Is everything that is certain determined in advance?

Is everything that is certain determined in advance?
Not if it is quantum.