Measuring the Thermodynamic Cost of Timekeeping

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Experiment and theory



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1. Is a clock a thermodynamic machine?

Clocks and thermodynamics

- Why clocks might be thermodynamic machines
- Theory of quantum clocks

2. Do clock signatures derived for the quantum regime still persist in classical systems?

Our experiment

- The optomechanical clock
- Results

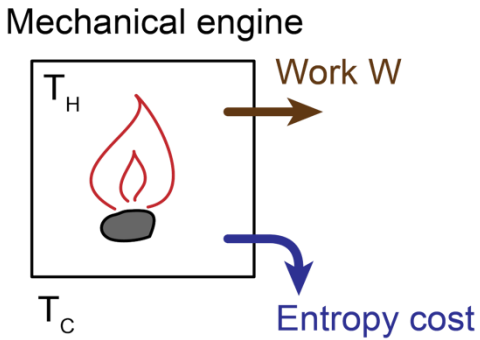
3. What are the implications?

Conclusions

Is a clock a thermodynamic machine?

Minimal thermodynamic costs

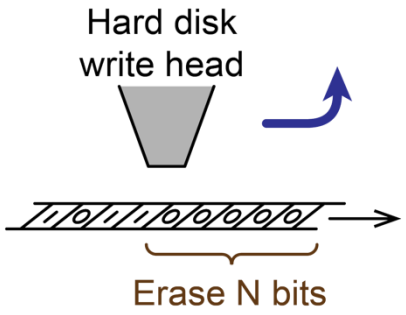
Converting heat to work



Entropy cost

$$\Delta S = \frac{W}{T_{\mathrm{H}} - T_{\mathrm{C}}}$$

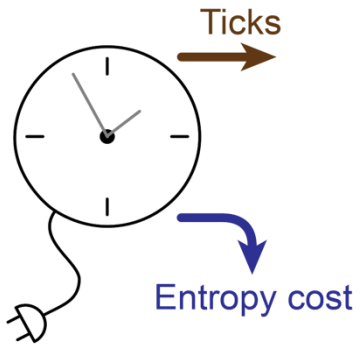
Erasing information



Entropy cost

$$\Delta S = N k_B \ln 2$$

Time keeping?



Entropy cost

$$\Delta S = ?$$

*diagrams by co-author E. Laird

Accuracy (N)

Let the accuracy of a clock (classical or quantum), be defined as:

$$N := \left(\frac{t_{\text{tick}}}{\Delta t_{\text{tick}}} \right)^2,$$

t_{tick} = the mean interval between ticks.

Δt_{tick} = standard deviation of the interval.

If successive tick intervals are uncorrelated, N is also the number of ticks before the expected accumulated timekeeping error is equal to one tick interval.

Classical vs quantum

Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031022 (2017)

Autonomous Quantum Clocks: Does Thermodynamics Limit Our Ability to Measure Time?

Paul Erker,[1,2] Mark T. Mitchison,[3,4] Ralph Silva,[5] Mischa P. Woods,[6,7] Nicolas Brunner,[5] and Marcus Huber[8]

(Received 7 November 2016; revised manuscript received 7 May 2017; published 2 August 2017)

Time remains one of the least well-understood concepts in physics, most notably in quantum mechanics. A central goal is to find the fundamental limits of measuring time. One of the main obstacles is the fact that time is not an observable and thus has to be measured indirectly. Here, we explore these questions by introducing a model of time measurements that is complete and autonomous. Specifically, our autonomous quantum clock consists of a system out of thermal equilibrium—a prerequisite for any system to function as a clock—powered by minimal resources, namely, two thermal baths at different temperatures. Through a detailed analysis of this specific clock model, we find that the laws of thermodynamics dictate a trade-off between the amount of dissipated heat and the clock's performance in terms of its accuracy and resolution. Our results furthermore imply that a fundamental entropy production is associated with the operation of *any* autonomous quantum clock, assuming that quantum machines cannot achieve perfect efficiency at finite power. More generally, autonomous clocks provide a natural framework for the exploration of fundamental questions about time in quantum theory and beyond.

The accuracy of an autonomous **quantum** clock is given by:

$$N_Q = \frac{\Delta S_{\text{tick}}}{2k_B}$$

We want: to see if this behaviour can be recognised in a classical system. In a simple nanoelectromechanical clock we will will identify an accuracy (N) and then compare it to a classical prediction, which we derive from the circuit electronics (N_C) and the quantum prediction (N_Q) above.

A (very) classical system



Consider a pendulum clock.

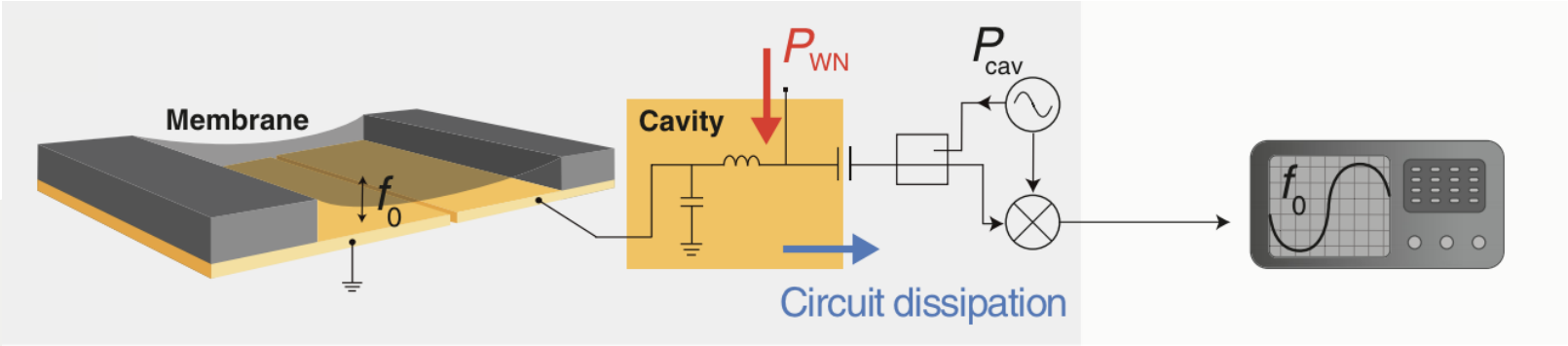
At first glance, there seems to be an inverse relationship between entropy and accuracy, since by decreasing the damping we can make the ticks more accurate and reduce the entropy cost.

However, this ignores the cost of observing the pendulum. Classically, there is no backaction from observation, at the nanoscale this changes and going to the quantum scales this changes even more profoundly.

Our experiment focuses on the nanoscale.

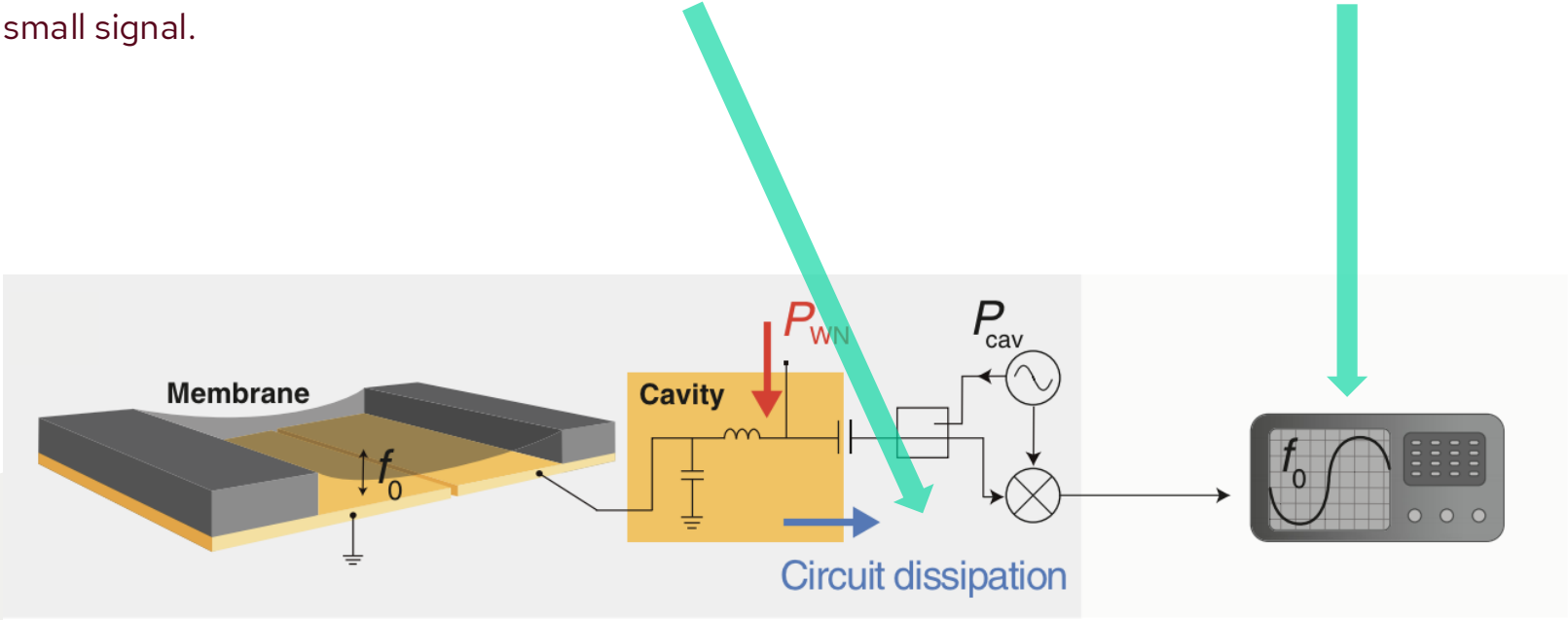
A nanoscale system: our setup

1. A nm thick membrane is driven by a white-noise signal with power P_{WN}.
2. The membrane's vibrations are probed by a rf cavity driven with a signal P_{cav} (illumination tone), which is capacitively coupled.
3. Part of this tone is reflected from the cavity, and its phase is modulated by the thermal vibration of the membrane.
4. The voltage output of the optomechanical readout circuit is proportional to the membrane displacement.



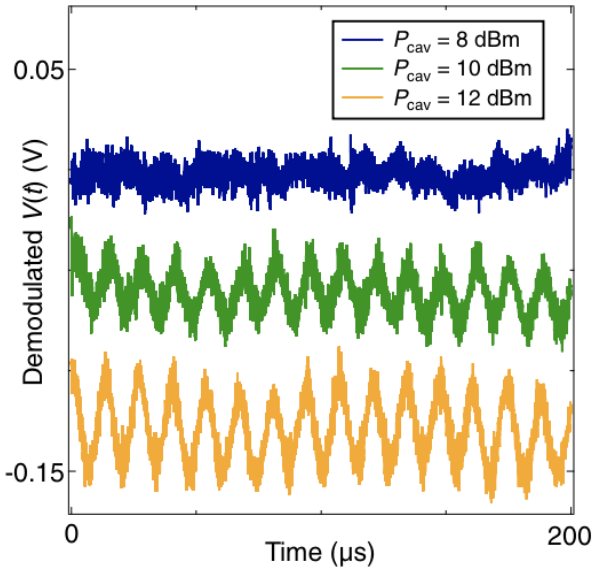
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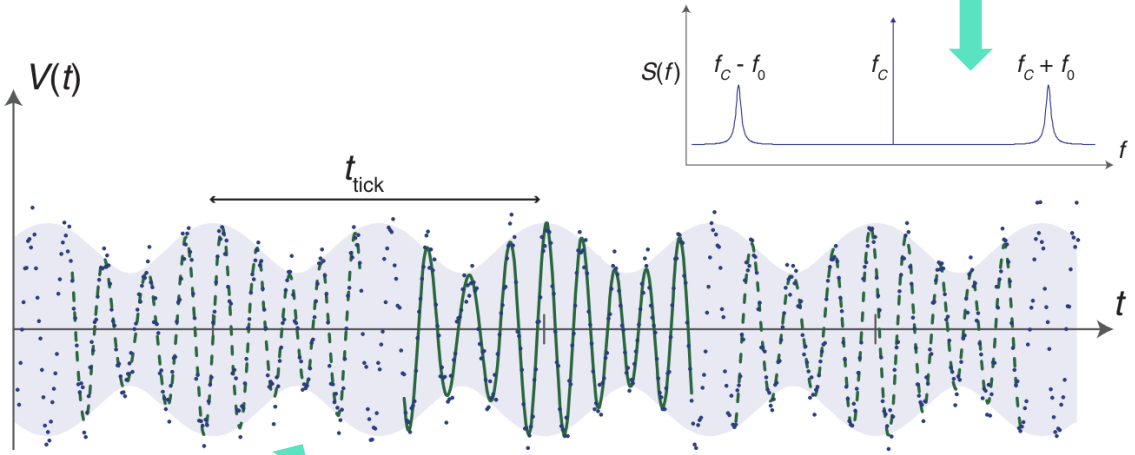
For accurate timekeeping, the modulated signal should be made as large as possible. This increases the dissipation in the detection circuit, and therefore creates greater entropy than a small signal.



Measured accuracy N: tick algorithm

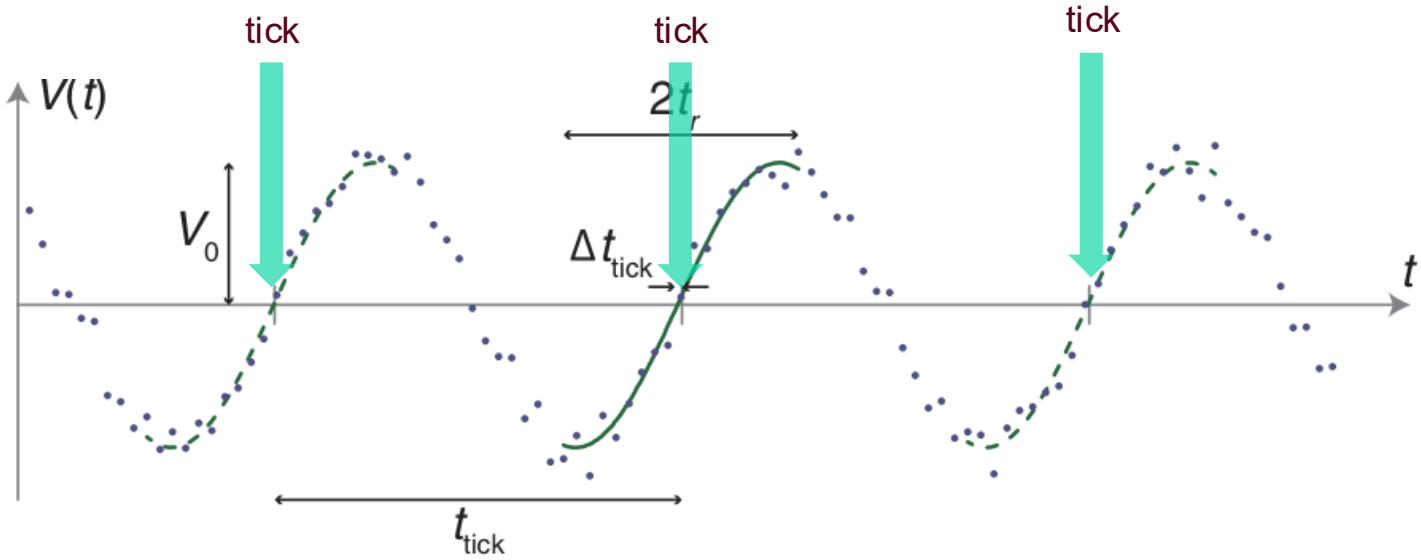
FT (power spectrum): uniform noise background, central peak from carrier, two thermomechanical sidebands. We work with circuit demodulated data.





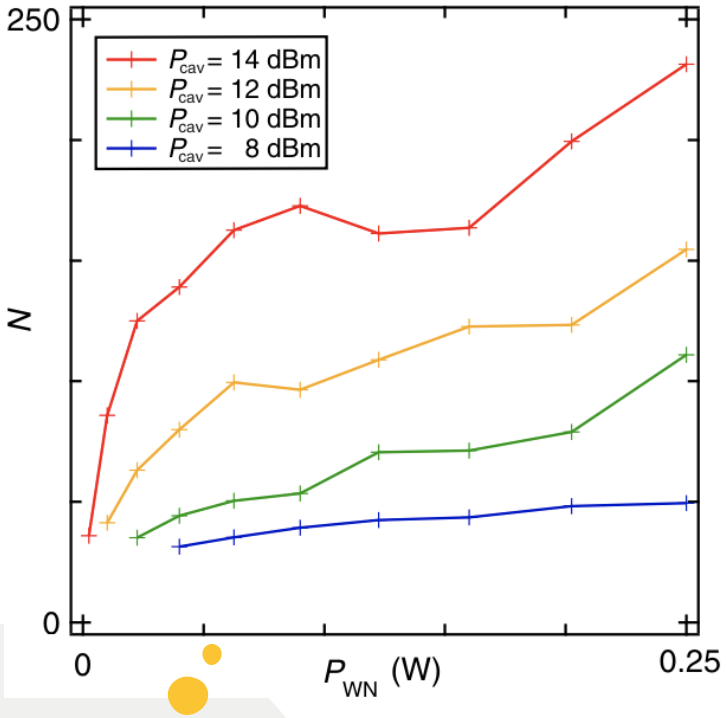
voltage record at the amplifier input

Measured accuracy N: tick algorithm



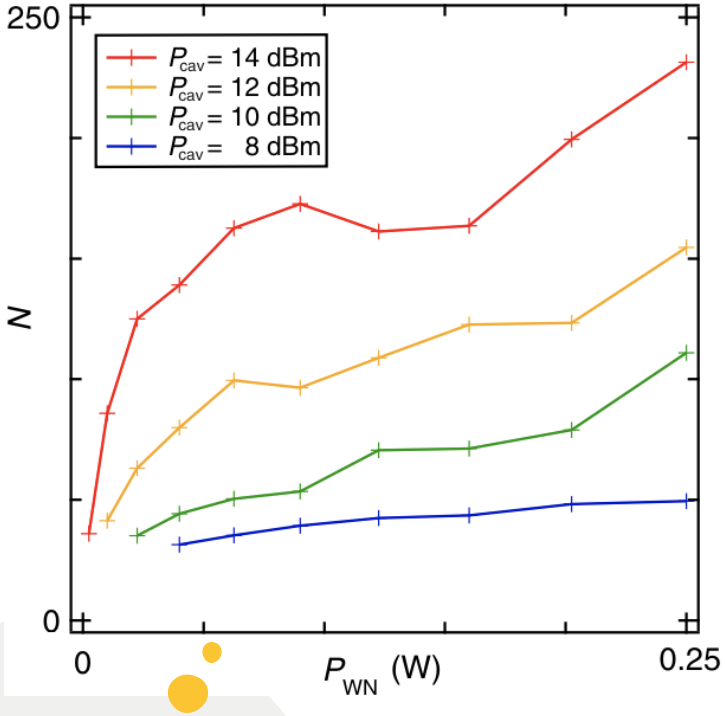
Tick identification algorithm must be "**on-the-fly**" otherwise not a clock.
Compute **mean** tick interval **and standard deviation** of this interval

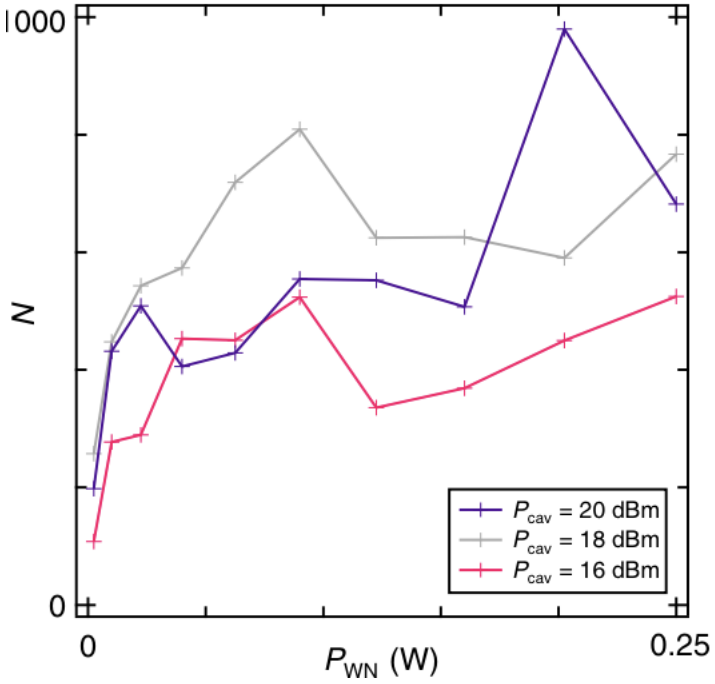
Results



- Plot of Accuracy (N) from tick algorithm vs white noise power P_{WN}
- Sampling rate of oscilloscope = 40 MSa/s, giving a resolution of 25. Upper limit to the measurable accuracy of $N \lesssim 290\,000$. Our values of N are less than a hundredth of this limit.
- Stronger driving makes the mechanical oscillations easier to distinguish from the noise
- An increased P_{cav} enhances the readout.

Results

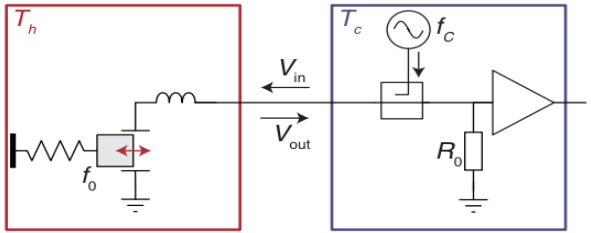

250
0
N
0
0.25
P_{WN} (W)
P_{cav} = 14 dBm
P_{cav} = 12 dBm
P_{cav} = 10 dBm
P_{cav} = 8 dBm


1000
0
N
0
0.25
P_{WN} (W)
P_{cav} = 20 dBm
P_{cav} = 18 dBm
P_{cav} = 16 dBm

How does N computed from the tick identification algorithm compare to the classical theoretical prediction N_C ?

Deriving accuracy N_C from circuit electronics

Optomechanical clock



LC tank circuit whose capacitance, is modulated by a thermo-mechanical resonator at temperature T_h. Tank circuit illuminated by a carrier tone V_{in} at freq f_c, giving rise to a reflected signal $V_{out,}$ which is passed to a cold matched resistor and amp at room temp T_c. The effect of the vibrations is to modulate the phase of V_{out}.

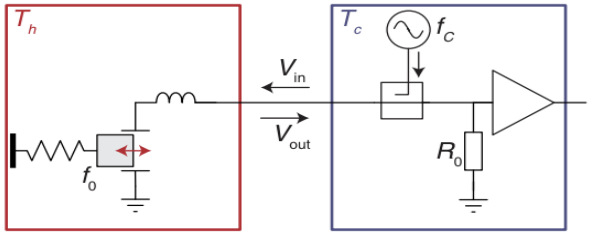
Ticks must be derived from a voltage record.

Q. How well can any clock identify a tick instant from a segment of this record?

A. Problem of phase estimation.

Deriving accuracy N_C from circuit electronics

Optomechanical clock

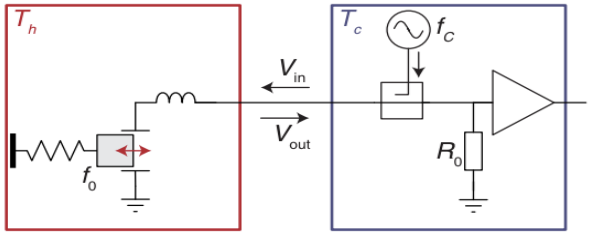


Entropy: created because the reflection from the tank circuit containing the hot resonator at T_h leads to irreversible heating in the cold resistor.

Heat contributions: (i) reflected carrier (coherent monochromatic tone at frequency f_c), (ii) two incoherent sidebands centred at $f_c \pm f_0$

Deriving accuracy N_C from circuit electronics

Optomechanical clock



$$\dot{S} = \frac{P_{SB}}{T_c}$$ S creation rate

$$N_C = \frac{2\pi^2}{k_B}\Delta S_{\text{tick}}$$ S per tick

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$$N_C = \frac{2\pi^2}{k_B}\frac{T_C}{T_N}\Delta S_{\text{tick}}$$

Assumption: amplifier noise << Johnson noise of the cold resistor. Other sources of noise contribute, and can be seen in the V record. Leads to decrease in N due to technical imperfections, rather than fund. bound.

Identifying the entropy in the experiment

Identify the entropy in the experiment and use it to compute $N_{C,}$ then compare with the measured N.

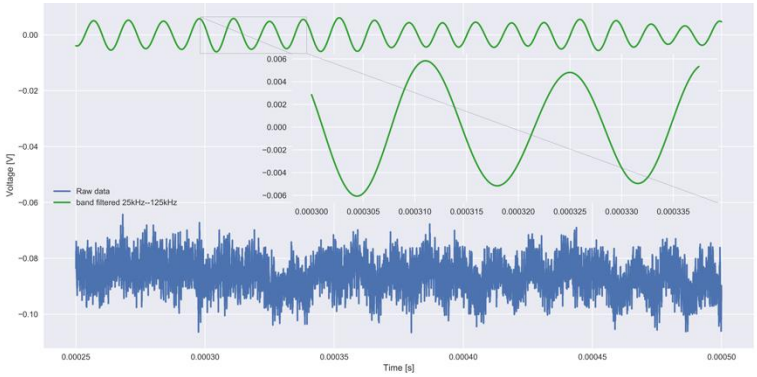
$$N_C = \frac{2\pi^2}{k_B}\frac{T_C}{T_N}\Delta S_{\text{tick}}$$

Focus: the entropy in the cavity output signal.

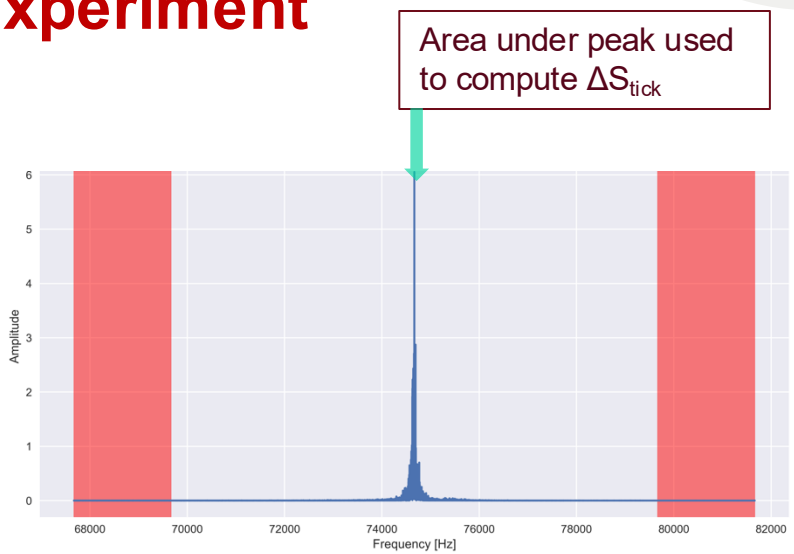
Other entropy contributions: (i) instruments used to control the systems (tone that drives the readout cavity, oscilloscope that measures the cavity output signal); (ii) white-noise driving.

Note: not all power injected into the system will be converted into useful signal, c.f. hot bath to heat engine work conversion. Ok, so long as power of the white-noise signal used to drive the clock is high enough to make the ticks identifiable above the thermal background.

Identifying the entropy in the experiment



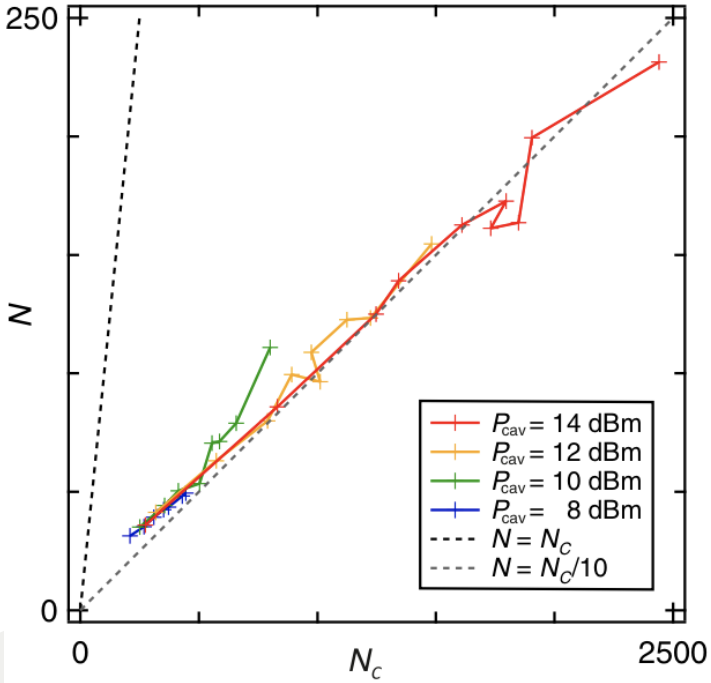
Fourier transform



Relevant entropy: arises from electrical power dissipated in the amp circuit by the optomechanical sidebands that contain the displacement information.

$$N_C = \frac{2\pi^2}{k_B}\frac{T_C}{T_N}\Delta S_{\text{tick}} = \frac{8\pi^2}{f_0}\frac{\int \mathcal{S}_{VV}^{(S)}(f)df}{\mathcal{S}_{VV}^{(N)}}$$

Measured accuracy (N) vs Accuracy from classical prediction (N_C)



- Our results confirm that **increasing accuracy requires increasing ΔS_{tick}** and show the linear relation predicted.
- The **constant of proportionality**, for all heating and illumination powers shown here, is approximately 10 times smaller than predicted.
- The classical **bound is not saturated** – need better tick identification algorithm.

Isn’t this just a measurement of a thermodynamic uncertainty relation?

Thermodynamic uncertainty relations (TURs)

The **“precision”** in the sense of a TUR is a statistical measure that characterizes the fluctuations of an observable F in an out-of-equilibrium system. It is defined as:

$$\mathfrak{p}(F) \equiv \frac{|\langle F \rangle|^2}{\mathrm{Var}(F)} \leq g(\dot{S})$$

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NB. These are not accuracy and precision as they are in metrology

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$$\mathfrak{p}(t_{\mathrm{tick}}) = \frac{(t_{\mathrm{tick}})^2}{(\Delta t_{\mathrm{tick}})^2} = N \leq 2\pi^2 t_{\mathrm{tick}} \frac{T_c}{T_N} \frac{\dot{S}}{k_B}$$

This is only useful if there is a mechanism to identify ticks and therefore correctly measure the observable. Naively applying p to the wrong observable, leads to a precision that behaves very differently to the one above.

NB. These are not accuracy and precision as they are in metrology

Thermodynamic uncertainty relations (TURs)

While the upper bound on precision increases with increased dissipated entropy, this **does not imply** an increased precision in more strongly driven systems.

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Higher entropy generation rate

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Higher entropy generation rate

Does not imply higher precision!

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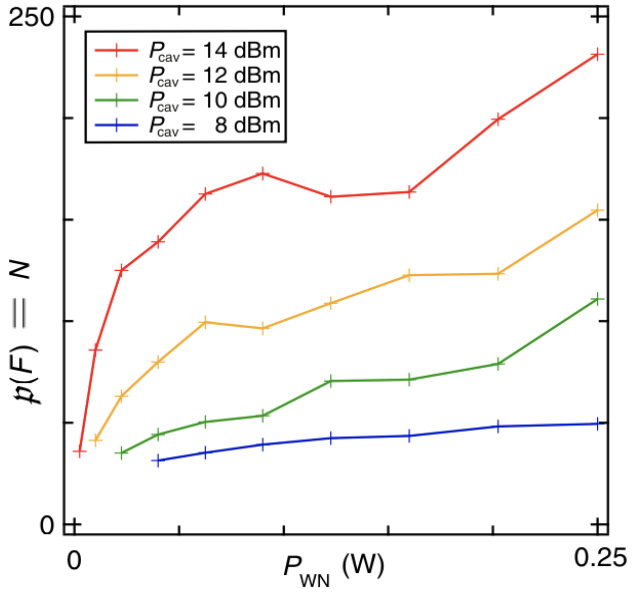
Higher entropy generation rate

Does not imply higher precision!

Argument: taking the 'wrong' observable for the precision in order to generate a TUR doesn't always lead to an increased precision with increasing power. With the wrong observable, running the clock at a higher entropy generation rate can actually lead to a worse precision!

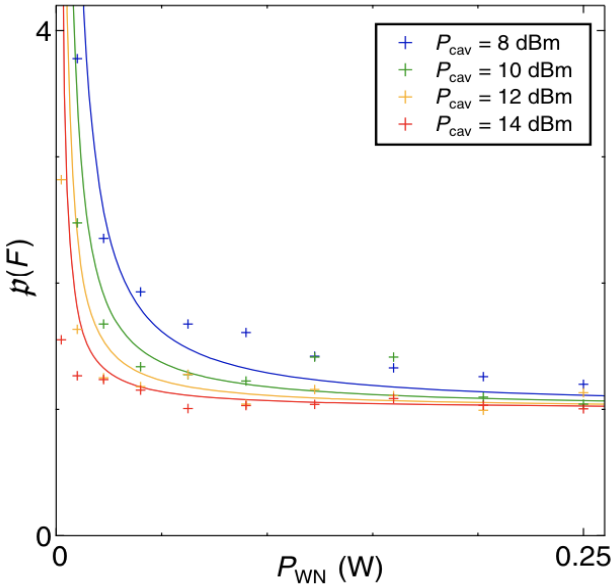
Example: Take **F** as the **energy flux** contained in the output of the clock, averaged over a fixed interval.

Thermodynamic uncertainty relations (TURs)



$$\langle F_{\rm tick} \rangle = t_{\rm tick} \qquad \mathfrak{p}(F_{\rm tick}) = N$$

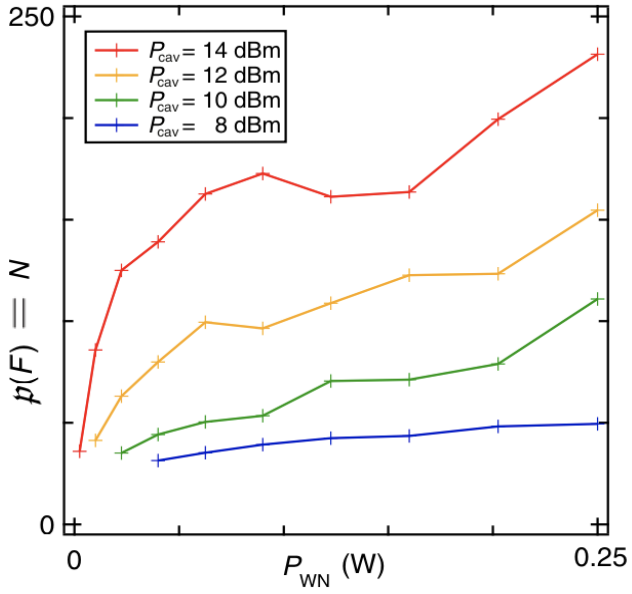
Noncanonical observable (ticks)



$$F(t) = \frac{1}{10t_{\rm tick}} \int_t^{t+10t_{\rm tick}} V^2(t')dt', \quad \mathfrak{p}(F) = \left(1 + \frac{\alpha}{P_{\rm cav}P_{\rm WN}}\right)^2$$

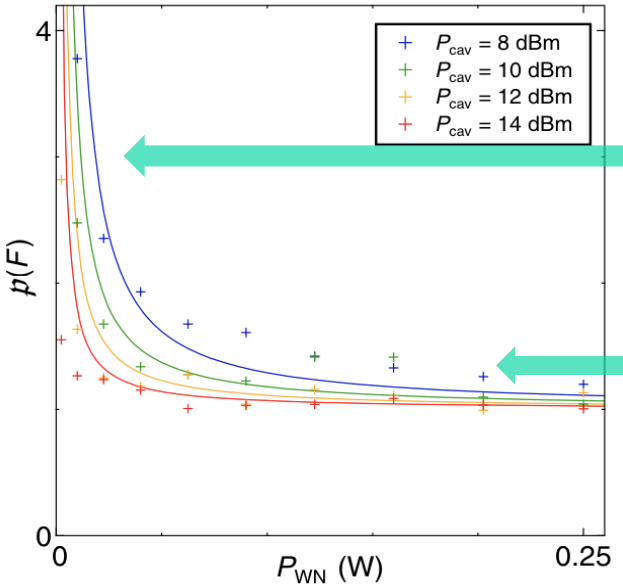
Conventional observable (energy flux)

Thermodynamic uncertainty relations (TURs)



$$\langle F_{\mathrm{tick}} \rangle = t_{\mathrm{tick}} \qquad \mathfrak{p}(F_{\mathrm{tick}}) = N$$

Noncanonical observable (ticks)



Noise dominates signal: white noise contains independent contributions to F at infinitely many frequencies.

Output dominated by the mechanical signal: in narrow frequency range but amplitude subject to long-lived fluctuations

$$F(t) = \frac{1}{10t_{\mathrm{tick}}} \int_t^{t+10t_{\mathrm{tick}}} V^2(t')dt', \quad \mathfrak{p}(F) = \left(1 + \frac{\alpha}{P_{\mathrm{cav}} P_{\mathrm{WN}}}\right)^2$$

Conventional observable (energy flux)

Conclusions

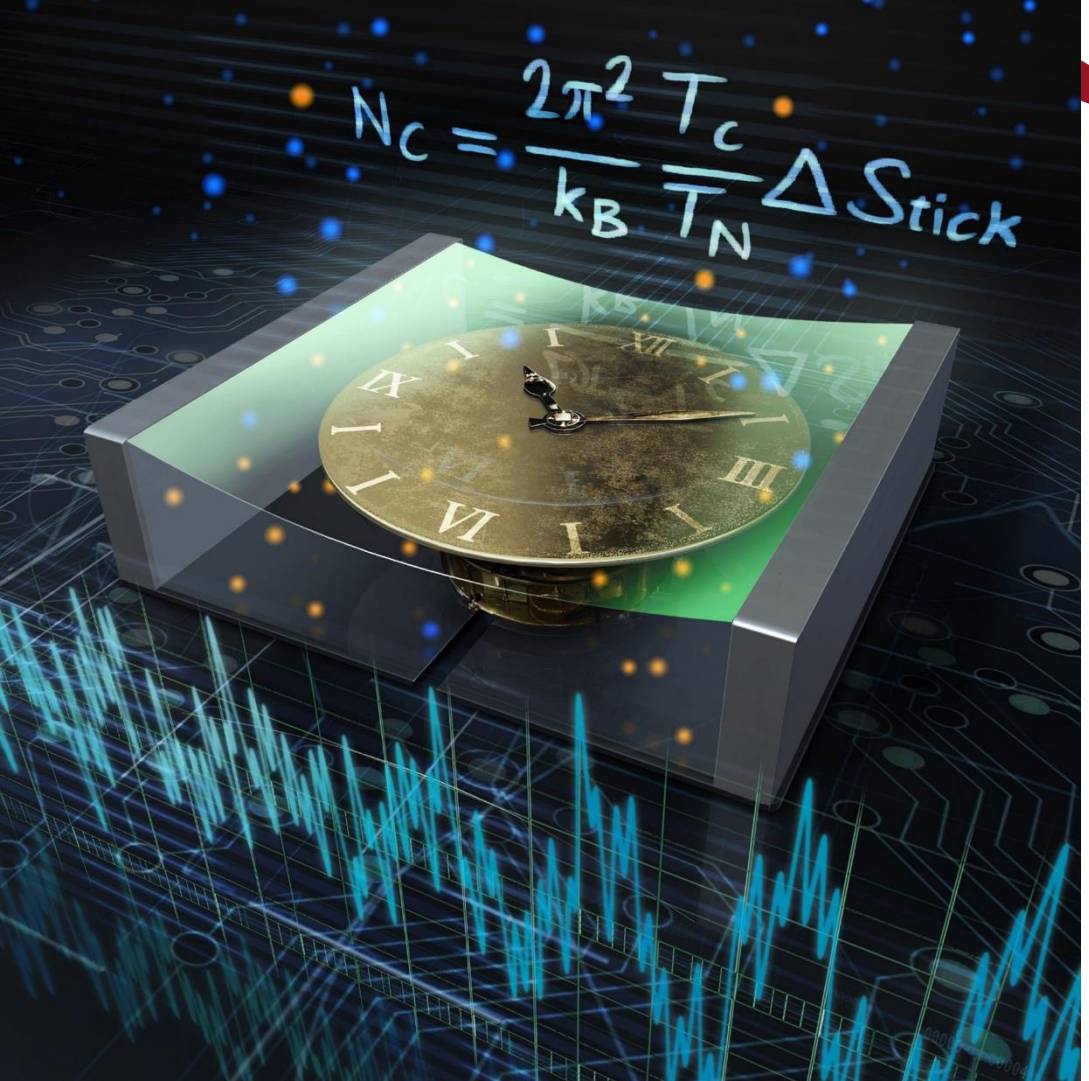
- The **heating** resource **enhances** the accuracy of the clock signal.
- In our system conventional thermodynamic currents do not experience an increased precision with increased driving.

Do clock signatures derived for the quantum regime still persist in classical systems?

- Yes. Signature: **entropy**, property: **accuracy**.
- Entropy dissipation is not only a prerequisite for measuring time, but that it is quantitatively related to the **fundamental limit** on a clock's performance.

Future work

- Clocks are fundamentally thermodynamic, what is their relation to TURs?
- Can noise (heat) from the environment be used to drive the system? in this way one would be able to say that the system is truly performing as a **useful engine.**



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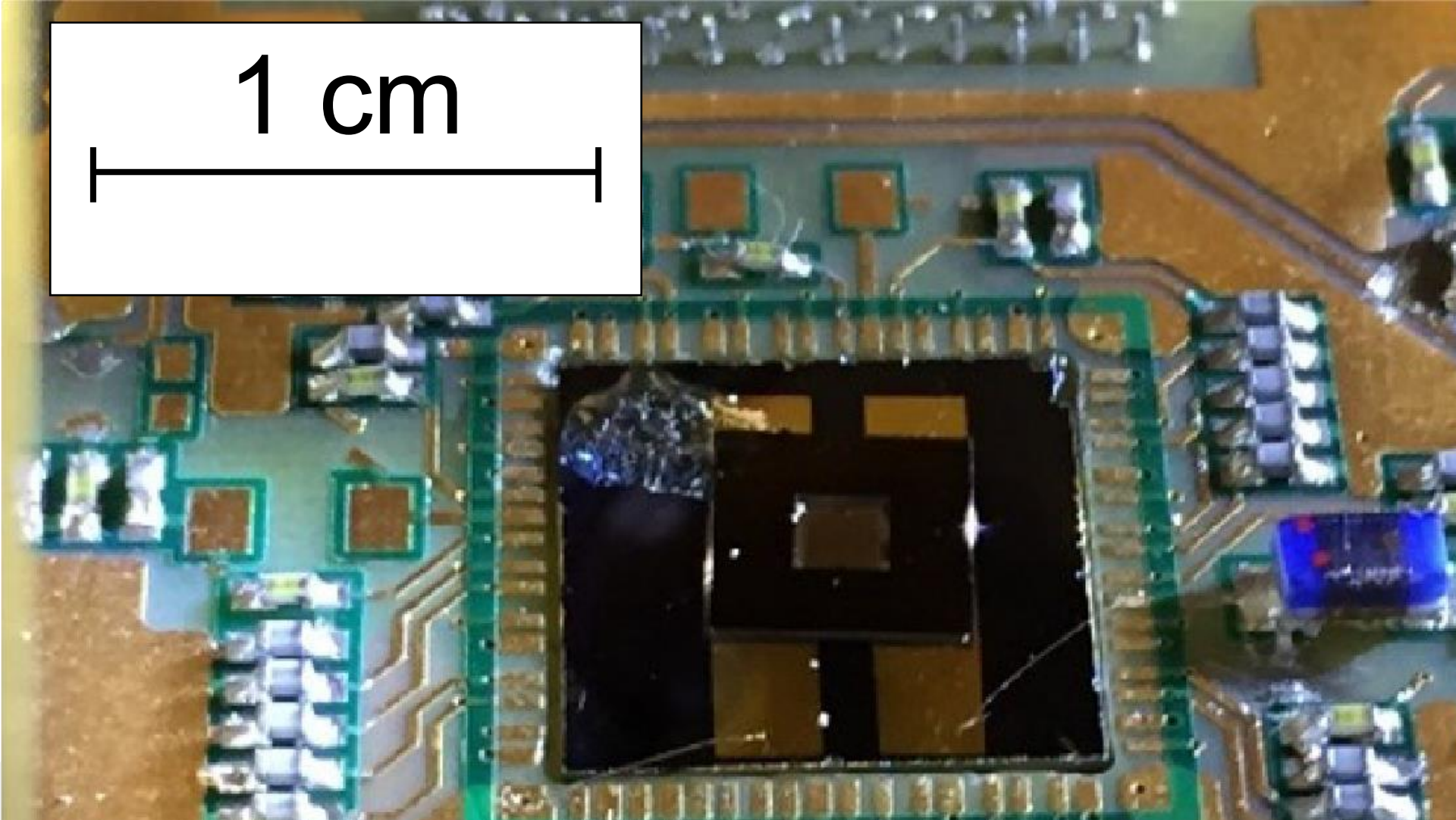
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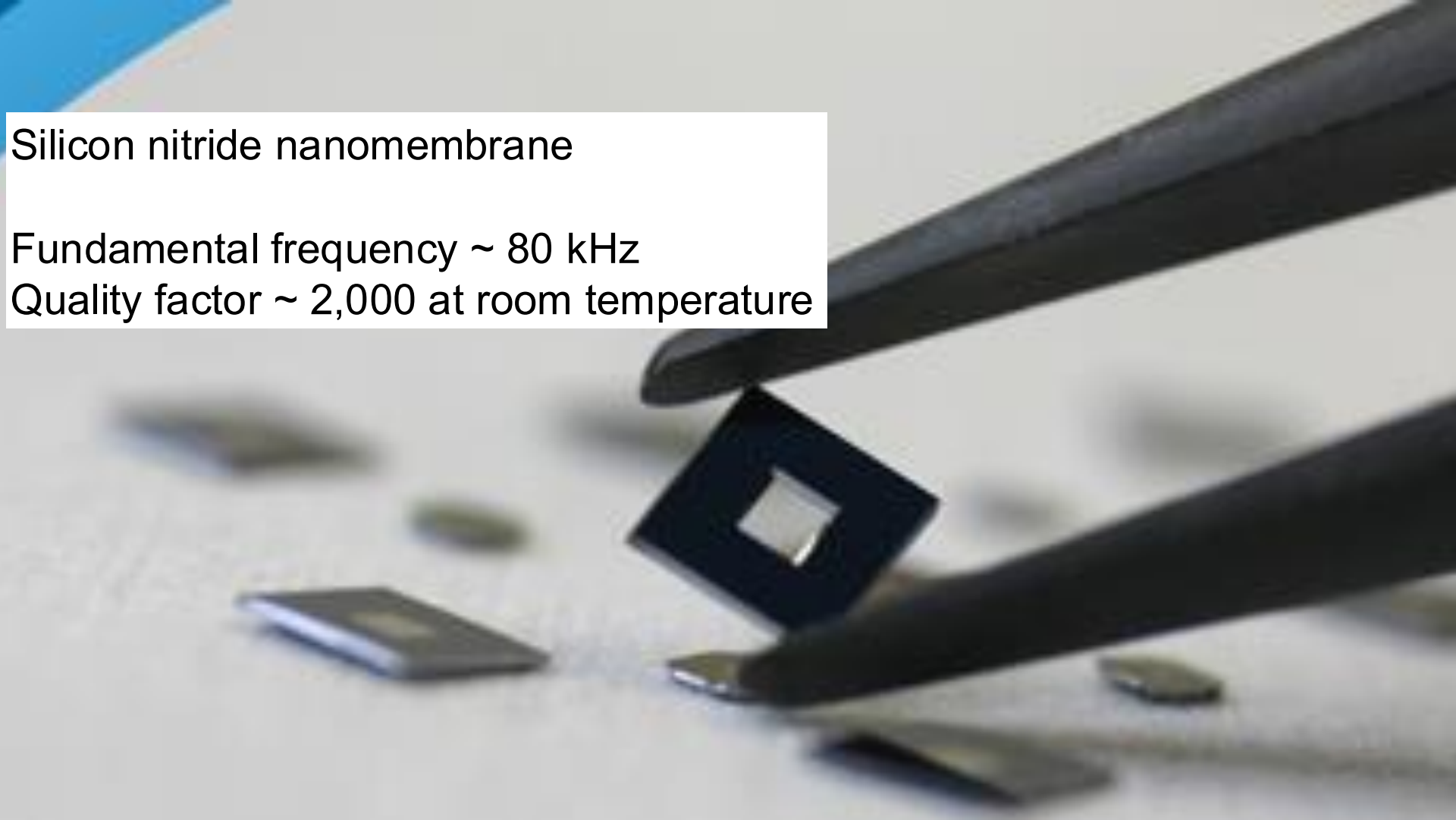
ÖAW
ÖSTERREICHISCHE AKADEMIE DER WISSENSCHAFTEN

TEMPLETON
World Charity Foundation, Inc.

Thank you.

Extra slides


1 cm


Silicon nitride nanomembrane
Fundamental frequency ~ 80 kHz
Quality factor ~ 2,000 at room temperature