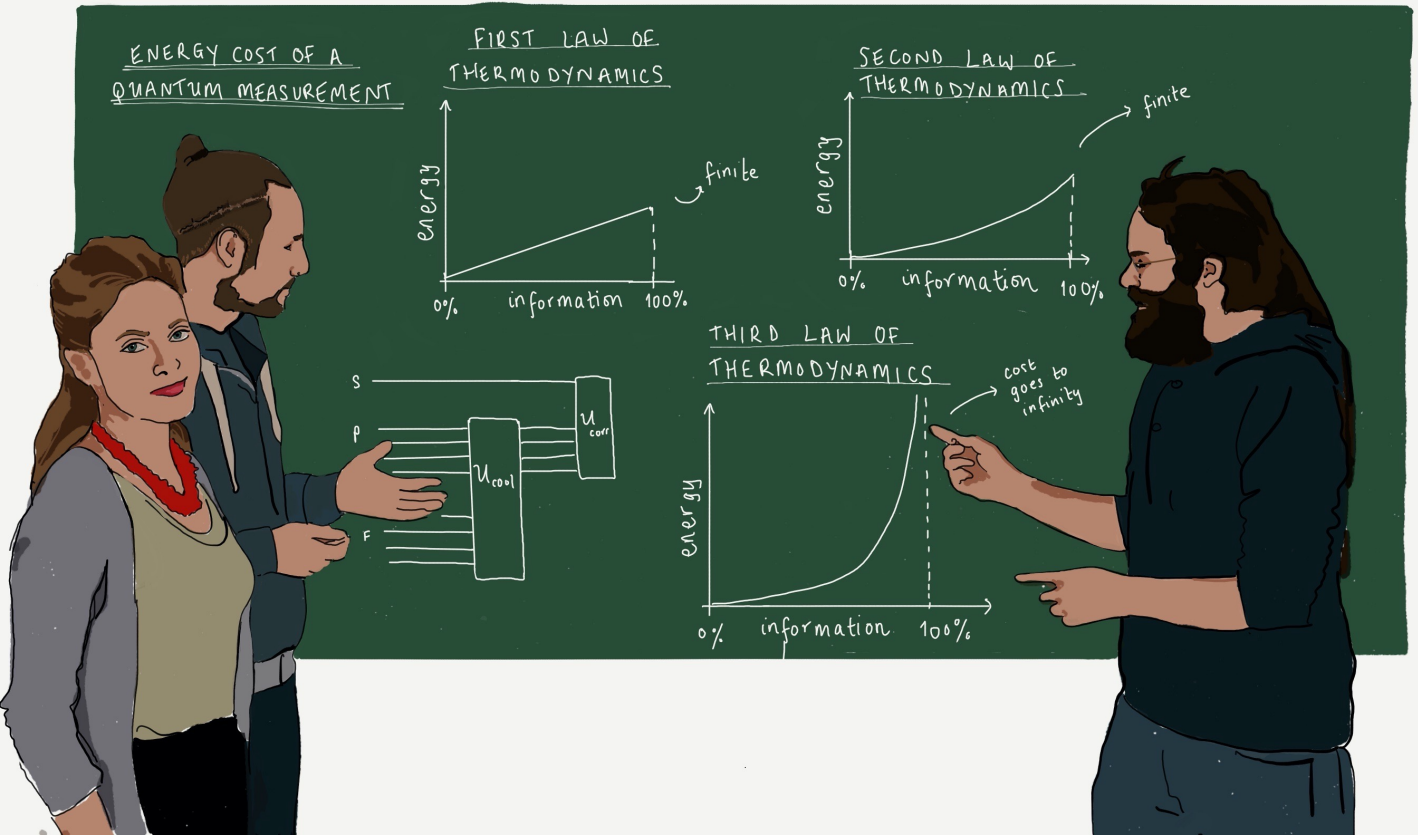


Ideal projective measurements have infinite resource costs

Yelena Guryanova
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APS March Meeting, Chicago, Illinois 2022.

Collaborators



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Ideal projective measurements have infinite resource costs
YG, Nicolai Friis and Marcus Huber
Quantum 4, 222, (2020)

Work estimation and work fluctuations in the presence of non-ideal measurements
Tiago Debarba, Gonzalo Manzano, YG, Marcus Huber, Nicolai Friis
New J. Phys. 21 113002 (2019)

Pure States and Projective Measurements – Convenient fictions in a thermodynamic world of incomplete knowledge

Nicolai Friis, 11.03.2022, Habilitation lecture

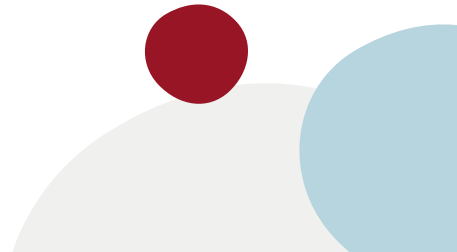
Quantum Measurements

Described by POVMs $\{M_i\}_i \quad \sum_i M_i^\dagger M_i = \mathbb{I}$

Post-measurement state $\rho_i' = \frac{M_i \rho M_i \dagger}{tr[M_i \rho M_i^\dagger]}$

Probability $tr[M_i \rho M_i^\dagger]$

Projective measurement $M_i^\dagger M_i = \Pi_i = |\psi_i\rangle \langle\psi_i|$



Thermodynamics from a quantum perspective

Global reversible time evolution

$$\rho(t) = U\rho_0 U^\dagger$$

Thermal states [12]

$$\tau(\beta) := \frac{e^{-\beta H}}{\mathcal{Z}(\beta)}$$

Unitary orbit of thermal states is full rank → third law [3]

[1] M. Gring, et. al, Relaxation and Pre-thermalization in an Isolated Quantum System, Science, Vol. 337 no. 6100 (2012)
[2] J. Eisert, M. Friesdorf & C. Gogolin, Quantum many-body systems out of equilibrium, Nature Physics volume 11, (2015)
[3] H. Wilming and R. Gallego, Third Law of Thermodynamics as a Single Inequality, Phys. Rev. X 7, 041033 (2017)

Thermodynamics from a quantum perspective

Global reversible time evolution

$$\rho(t) = U\rho_0 U^\dagger$$

Emergence of thermal states through entanglement and course graining [12]

$$\tau(\beta) := \frac{e^{-\beta H}}{\mathcal{Z}(\beta)}$$

Unitary orbit of thermal states is full rank → third law [3]

Unattainability principle: it is impossible to cool down to absolute zero with finite resources.

Thermodynamics from a quantum perspective



Cooling a state

- Fridge coupled to target system directly (laser cooling), or bulk (dilution, etc.)
- Work invested, target system gets closer to ground state
- Limited by third law

Thermodynamics from a quantum perspective



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Measuring a state

- Measurement apparatus coupled to target system.
- Measurement in energy eigenbasis. Outcome $|E_i\rangle\langle E_i|$ with probability $\frac{e^{-\beta E_i}}{\mathcal{Z}(\beta)}$
- Limitations?

Contradiction ?



Cooling a state

- Fridge coupled to target system directly (laser cooling), or bulk (dilution, etc.)
- Work invested, target system gets closer to ground state
- Limited by third law

PURE STATE



Measuring a state

- Measurement apparatus coupled to target system.
- Measurement in energy eigenbasis. Outcome $|E_i\rangle\langle E_i|$ with probability
- Limitations?

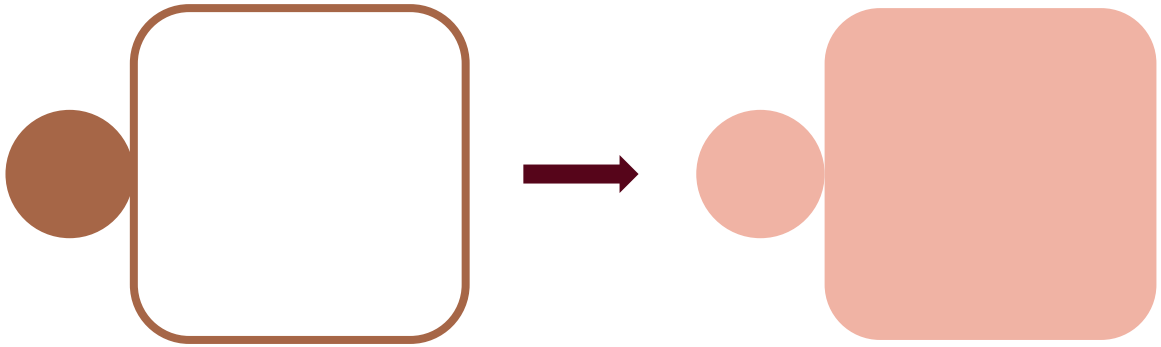
PURE STATE

Outline

- What is a quantum measurement?
- Ideal measurement
- Idea measurements are not possible
- Applications and corrections
- Summary

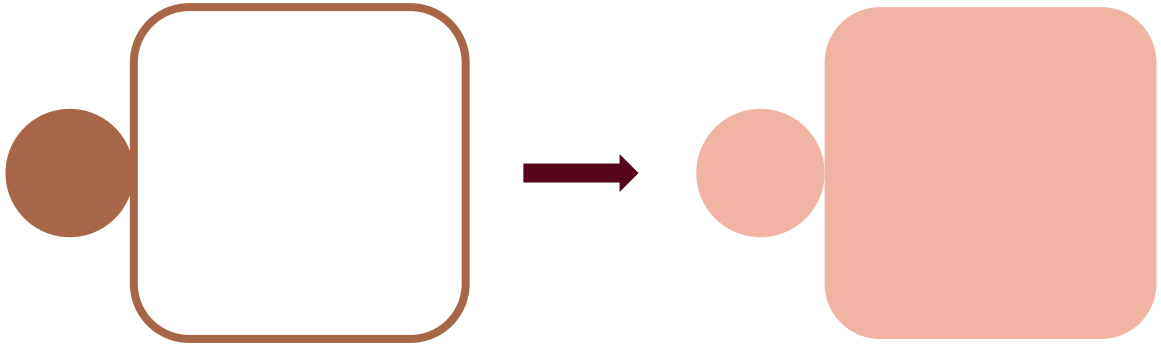
What is a measurement?

$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$



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$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$

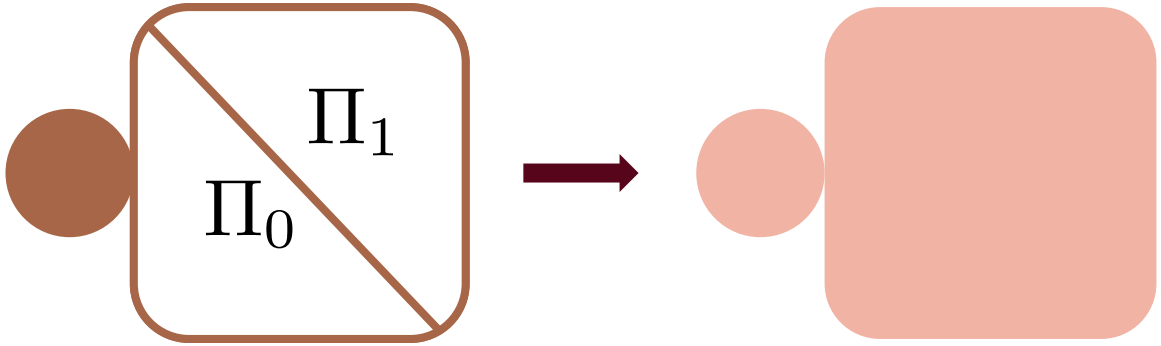


$$|0\rangle_S \longleftrightarrow \Pi_0$$
$$|1\rangle_S \longleftrightarrow \Pi_1$$

$$\Pi_i = \sum_n |\tilde{\psi}_n^{(i)}\rangle\langle\tilde{\psi}_n^{(i)}|$$

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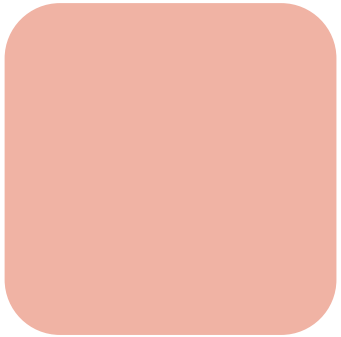


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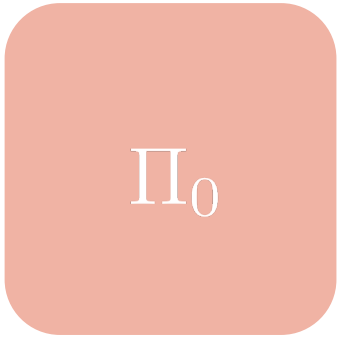
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"observing the pointer" $\tilde{\rho}_P$

What is a measurement?

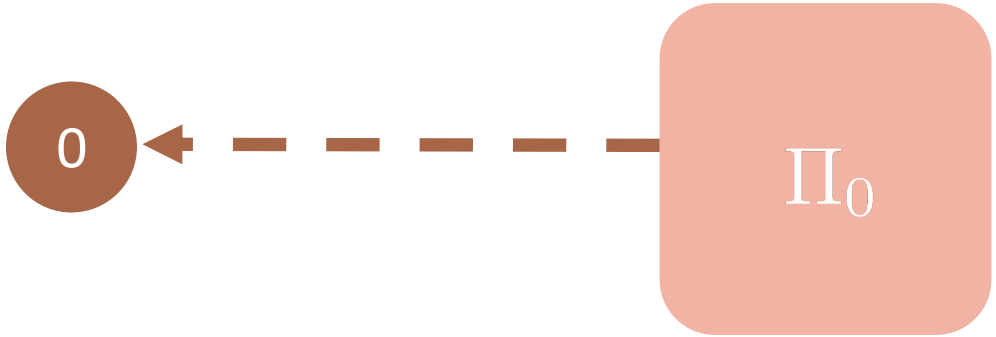
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What is a measurement?

$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$



"observing the pointer" $\tilde{\rho}_P$

Ideal measurement: 3 properties

Unbiased: "Pointer and system statistics are the same"

$$\begin{aligned}\mathrm{tr}\left[\mathbb{I}\otimes\ \Pi_i\tilde{\rho}_{SP}\right] &= \mathrm{tr}\left[\ |i\rangle\langle i|_S\ \rho_S\right] \\ &= \rho_{ii} \quad \forall i.\end{aligned}$$

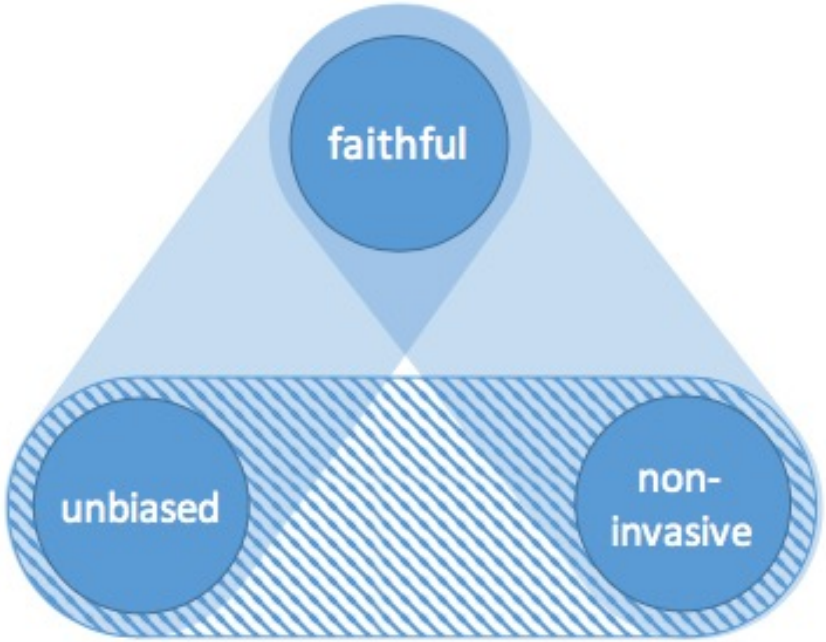
Faithful: "the probability that your guess is correct"

$$C(\tilde{\rho}_{SP}) := \sum_i \mathrm{tr}\left[\ |i\rangle\langle i|\otimes\Pi_i\ \tilde{\rho}_{SP}\right] = 1$$

Non-invasive: "the system is not disturbed on the diagonal.

$$\begin{aligned}\mathrm{tr}\left[\ |i\rangle\langle i|_S\ \tilde{\rho}_S\right] &= \mathrm{tr}\left[\ |i\rangle\langle i|_S\ \rho_S\right] \\ &= \rho_{ii} \quad \forall\ i.\end{aligned}$$

Ideal measurement: 3 properties are interrelated



The three properties of an ideal measurement are related pairwise: any pair of properties implies the third*. If one of them does not hold then the triangle collapses.

Ideal measurement: example

Example: image your measurement leaves your system and pointer in the state $|00\rangle\langle 00|_{SP}$ .This pure, classically correlated, product state has perfect correlation. The measurement that resulted in this state is said to be faithful.

$$C(|00\rangle\langle 00|_{SP}) = \mathrm{tr}\left[\left(|0\rangle\langle 0| \otimes \Pi_0 + |1\rangle\langle 1| \otimes \Pi_1\right) \ |00\rangle\langle 00|_{SP}\right]$$
$$= \ 1$$

Ideal measurement: qubit example

An ideal measurement of a qubit system by a 1-qubit pointer.

$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Ideal measurement: qubit example

An ideal measurement of a qubit system by a 1-qubit pointer.

$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$

$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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U_{CNOT}

Unbiased: "Pointer and system statistics are the same"

Faithful: "the probability that your guess is correct"

Non-invasive: "the system is not disturbed on the diagonal.

Ideal measurement: any dimension

An ideal measurement of *any* dimensional system by a 1-qubit pointer.

$$\tilde{\rho}_{SP} = U_{\mathrm{CNOT}} \left(\rho_S \otimes |0\rangle\langle 0|_P\right) U_{\mathrm{CNOT}}^{\dagger}$$

$$U_{CNOT} := |0\rangle\langle 0|_S \otimes \mathbb{1}_P + \sum_{i\neq 0} |i\rangle\langle i|_S \otimes X_P^{(i)}$$

$$X_P^{(i)} = |0\rangle\langle i| + |i\rangle\langle 0| + \sum_{j\neq 0,i} |j\rangle\langle j|$$

Unbiased: "Pointer and system statistics are the same"

Faithful: "the probability that your guess is correct"

Non-invasive: "the system is not disturbed on the diagonal.

Non-ideal measurement

A measurement is non-ideal if any one of the 3 properties fails to hold.

Unbiased: "Pointer and system statistics are the same"

Faithful: "the probability that your guess is correct"

Non-invasive: "the system is not disturbed on the diagonal.

Theorem: ideal measurements are not possible

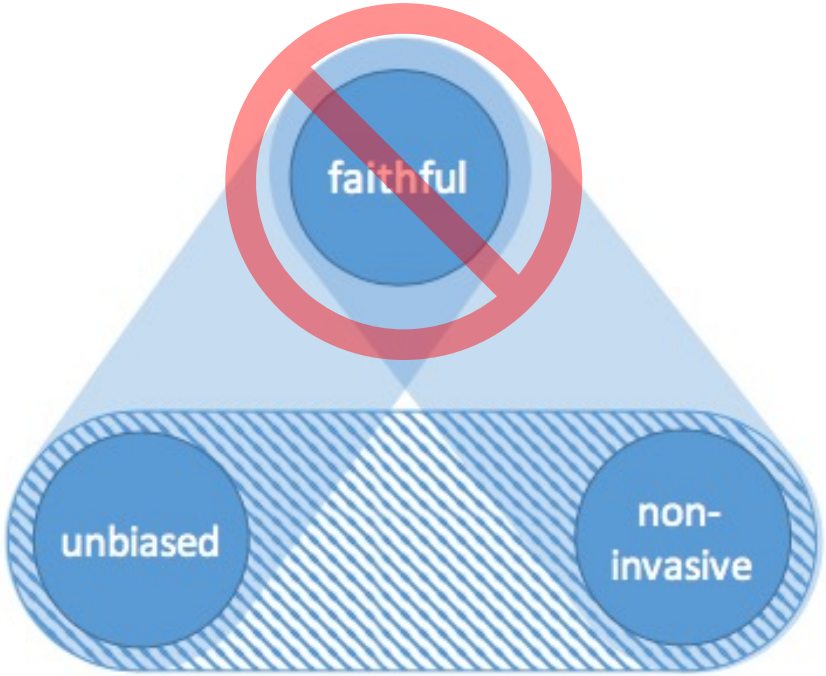
Faithful measurements (perfect correlations) are possible **if and only if** one can prepare pure states.

By the third law of thermodynamics, one cannot prepare pure states, therefore **faithful** measurements not possible → ideal measurements are not physically feasible.

L. Masanes and J. Oppenheim, "A general derivation and quantification of the third law of thermodynamics," Nat. Commun. 8, 14538 (2017), arXiv:1412.3828.
L. J. Schulman, T. Mor, and Y. Weinstein, "Physical Limits of Heat-Bath Algorithmic Cooling," Phys. Rev. Lett. 94, 120501 (2005). H. Wilming and R. Gallego, "Third Law of Thermodynamics as a Single Inequality," Phys Rev. X 7, 041033 (2017), arXiv:1701.07478.
F. Clivaz, R. Silva, G. Haack, J. Bohr Brask, N. Brunner, and M. Huber, "Unifying paradigms of quantum refrigeration: resource-dependent limits," (2017), arXiv:1710.11624.
J. Scharlau and M. P. Mu¨ller, "Quantum Horn's lemma, finite heat baths, and the third law of thermodynamics," Quantum 2, 54 (2018), arXiv:1605.06092.

Creating pure states costs infinite energy, takes infinite time, or requires infinite particles.

Approximating ideal measurements



The three properties of an ideal measurement are related pairwise: any pair of properties implies the third*. If one of them does not hold then the triangle collapses.

Approximating ideal measurements

Since ideal measurements are not possible, we would like to determine how closely they can be approximated.

The initial pointer state must be full rank to avoid self-referential descriptions of measurements which rely on pure states

$$\rho_P = \tau_P(\beta) = \frac{1}{\mathcal{Z}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-\beta E_P} \end{pmatrix}$$

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$$\rho_S \otimes \rho_P \longrightarrow \tilde{\rho}_{SP}$$ before

$$U_{\text{tot}} \left(\rho_S \otimes \tau_P(\beta)\right) U_{\text{tot}}^{\dagger} = \tilde{\rho}_{SP}$$ now

$$\rho_P = \tau_P(\beta) = \frac{1}{\mathcal{Z}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-\beta E_P} \end{pmatrix}$$

1. Unbiased
2. Faithful
3. Non-invasive

Approximating ideal measurements

Goal: to bring $\tilde{\rho}_{SP}$ as close as possible to the ideal post-interaction state. We know that pure states make the best pointers.

1. **Cool** the pointer (bring it closer to a pure state).

2. **Correlate** the pointer and system (generate as much correlation as possible to effect the measurement).

1. Unbiased ✓
2. Faithful ✗
3. Non-invasive ✗

Approximating ideal measurements

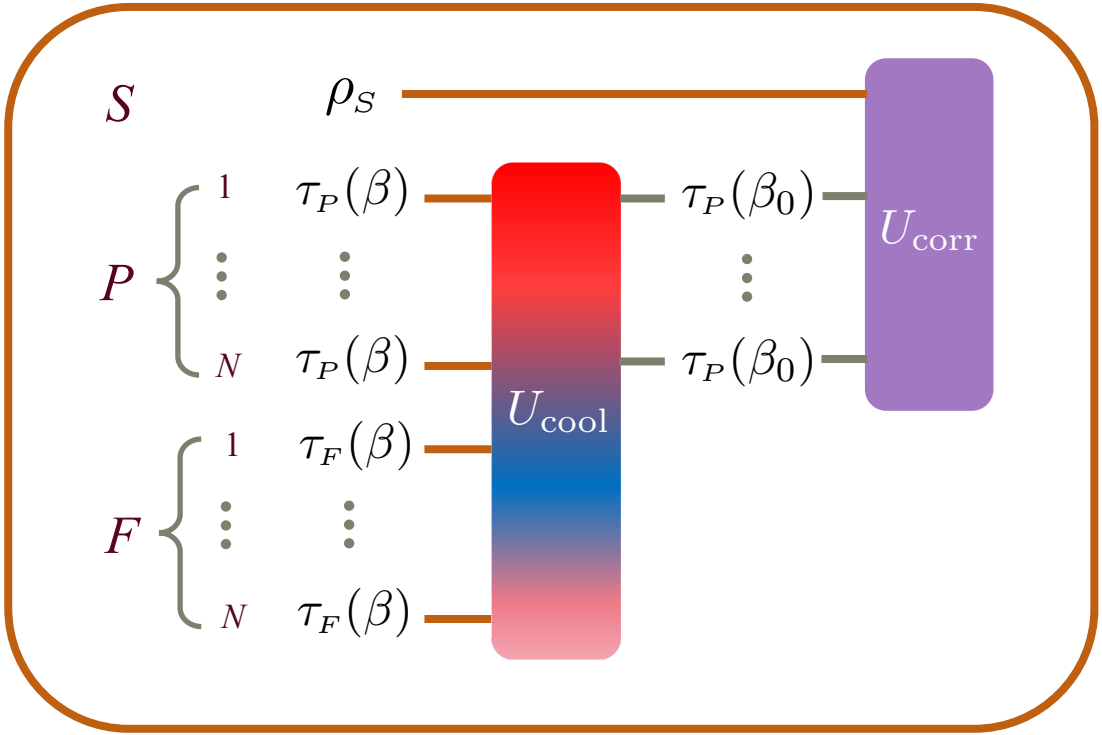
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U_{tot} {

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Schematic


S
ρ_S
P
1
N
$\tau_P(\beta)$
$\tau_P(\beta)$
F
1
N
$\tau_F(\beta)$
$\tau_F(\beta)$
U_{cool}
$\tau_P(\beta_0)$
$\tau_P(\beta_0)$
U_{corr}

Energy cost of unbiased measurements

$$\Delta E = \mathrm{tr}[(H_S + H_P)(\tilde{\rho}_{SP} - \rho_{SP})]$$
$$= \Delta E_{\mathrm{cool}} + \Delta E_{\mathrm{corr}}$$

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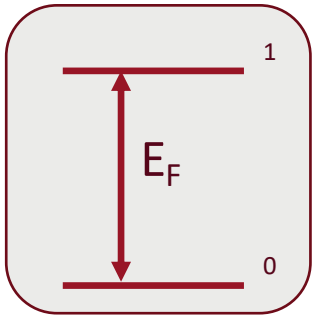
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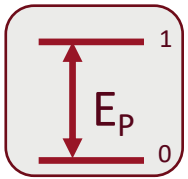
$$\Delta E = \mathrm{tr}[(H_S + H_P)(\tilde{\rho}_{SP} - \rho_{SP})]$$
$$= \Delta E_{\mathrm{cool}} + \Delta E_{\mathrm{corr}}$$

1. **Cool** the pointer, e.g. by using the 1-qubit fridge [1].

$$\tau_P(\beta)^{\otimes N} \mapsto \tau_P(\beta \tfrac{E_F}{E_P})^{\otimes N}$$

$$\Delta E_{\mathrm{cool}} = N(E_F - 1)\left(\frac{1}{e^{-\beta E_F} + 1} - \frac{1}{e^{-\beta E_P} + 1}\right)$$





[1] F. Clivaz, R. Silva, G. Haack, J. Bohr Brask, N. Brunner, and M. Huber, "Unifying paradigms of quantum refrigeration: resource-dependent limits," Phys. Rev. E **100**, 042130 Oct. 2019

Energy cost of unbiased measurements

$$\Delta E = \mathrm{tr}[(H_S + H_P)(\tilde{\rho}_{SP} - \rho_{SP})]$$
$$= \Delta E_{\mathrm{cool}} + \Delta E_{\mathrm{corr}}$$

2. **Correlate** the pointer and system.

$$\min_{U_{\mathrm{corr}}} \Delta E_{\mathrm{corr}} \quad \text{s.t.} \quad C(\tilde{\rho}_{SP}) = C_{\max}(\beta_0, N)$$

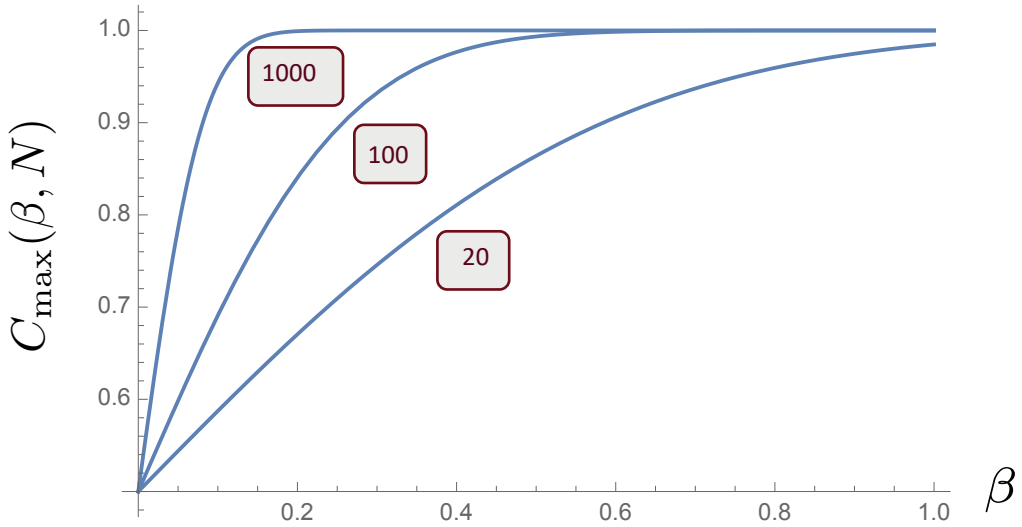
analytic

$$C_{\max}(\beta, N) = \frac{1}{\mathcal{Z}^N} \sum_{k=0}^{N/2} \binom{N}{k} e^{-kE_P\beta}.$$

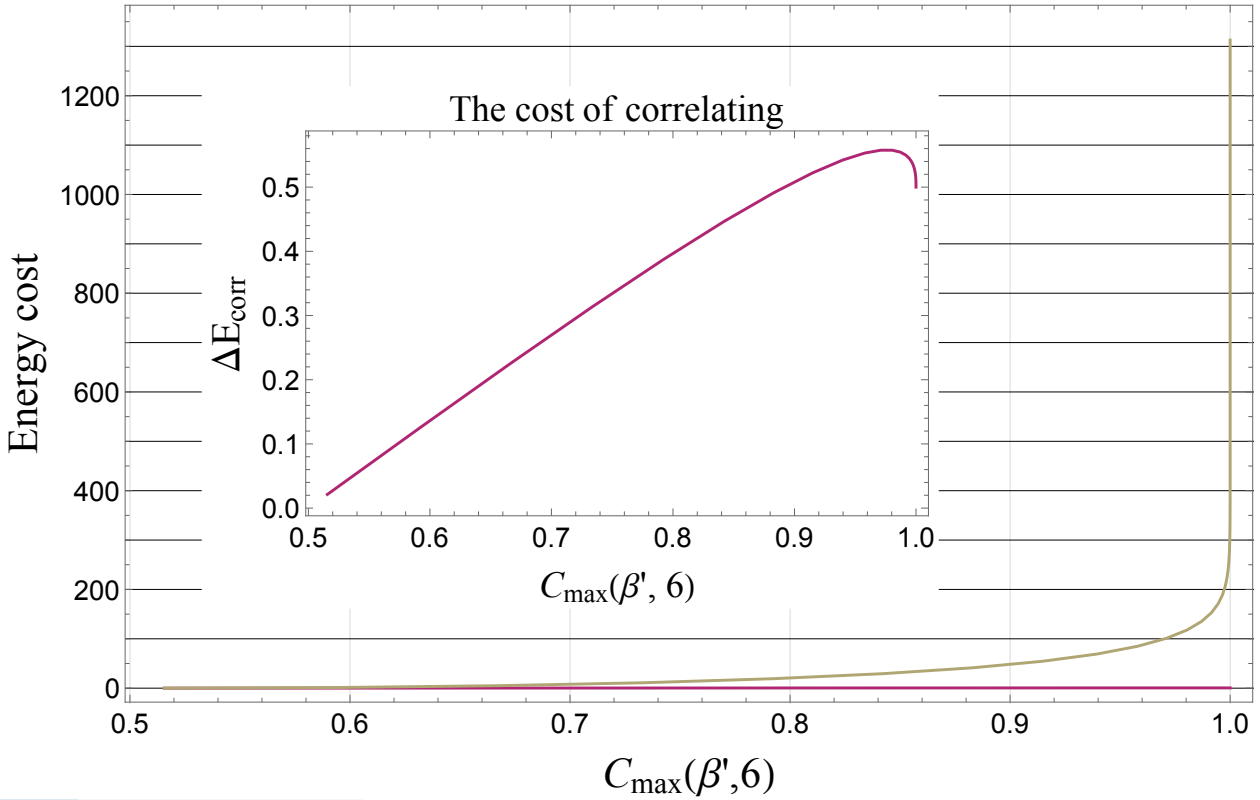
For pointers that start in a thermal state, C_{max} depends on the temperature, spectral gaps and dimension (no. of particles).

Best correlations for an N-qubit pointer

$$C_{\max}(\beta, N) = \frac{1}{\mathcal{Z}^N} \sum_{k=0}^{N/2} \binom{N}{k} e^{-k E_P \beta} .$$



Total cost of an ideal measurement



1. **System:** 1 qubit, **Pointer:** 6-qubits (energy gap in microwave regime)
2. Start at room temp and cool with a fridge with gap E_F , until the fridge is "exhausted".
3. Correlate from new temp **β'.**

Summary so far

Theorem: an unbiased measurement (system and pointer statistics the same) can always be implemented at finite cost and in finite time.

Corollary: to be faithful (perfect correlation, pointer never makes a mistake) the pointer has to be cooled, subject to the laws of thermodynamics.

Consequence: obtaining reliable knowledge about quantum systems generically requires macroscopic quantities of work; perfect knowledge is infinitely expensive.

Zero entropy states can only be prepared asymptotically

Work Estimation and Work Fluctuations in the Presence of Non-Ideal Measurements

- Obtaining estimates with a finite amount of work implies an imperfection in the estimates themselves.
- Energy delivered by measurements is not free of charge and must be supplied to realise the measurement in the first place.

Jarzynski equality and Crooks' relation

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

Relates free energy differences to the work that is applied to the system in order to drive the system from one state to another (beginning to end).

$$P_{\mathrm{B}}(-W) = e^{-\beta(W-\Delta F)} P_{\mathrm{F}}(W)$$

This relates the probability $P_{\mathrm{F}}(W)$ of performing some work during a realisation of the TPM scheme and $P_{\mathrm{B}}(-W)$ for extracting the same amount of work when the time-reversed protocol is implemented.

Jarzynski equality and Crooks' relation

$$\langle e^{-\beta W} \rangle_{\text{non-id}} = \chi\, e^{-\beta \Delta F}$$

The invasiveness of the first non-ideal measurement spoils the requirement that the system starts the process in equilibrium. Additional energetic cost only depends on the system Hamiltonian (not the pointer Hamiltonian).

$$P_{\text{B}}(-W) = e^{-\beta(W-\Delta F)} P_{\text{F}}(W)$$

Both non-ideal measurements act as (independent) noise sources, disturbing the initial states of the forward and backward TPM processes, respectively

Conclusion

Pure states are not for free.

- Experimentalists understand well that perfect projective measurements are not exactly possible but are very well approximated using their high-power consumption equipment (APDs and Nanowire detectors for photons, lasers and CCDs for ion traps and cold atoms).
- Past approaches typically assumed that projective measurements can be carried out perfectly and that their cost can be attributed to the work value of the measurement outcome. Others adopt the stance that Landauer erasure bound represents the cost of resetting devices to pure states without providing conclusive evidence that the bound is achievable.
- In thermal environments, measurements produce errors, which can be mitigated by either reducing the temperature of the environment, or by using larger measuring devices.

Collaborators



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Tiago de Barba
Federal University of
Technology, Paraná, Brazil.

Thank you

Universitat de les Illes Balears

TU WIEN

UTFPR

IQOQI

Extra slides.

Ideal measurement: special form

The **ideal post-interaction state** with perfect correlation has the form

$$\tilde{\rho}_{SP} = \sum_i \rho_{ii} \, |i\rangle\langle i| \otimes \rho^{(i)} + \text{off-diag.},$$

i.e. perfect correlations **iff** the post-interaction state has this form

system

pointer

$$\Pi_i \rho^{(j)} = \delta_{ij} \rho^{(j)}$$

Two-point measurement (TPM) scheme (ideal case)

$$(\tau^{(0)}, H^{(0)}) \xrightarrow{\Lambda} (\rho^{(\mathrm{f})}, H^{(\mathrm{f})})$$

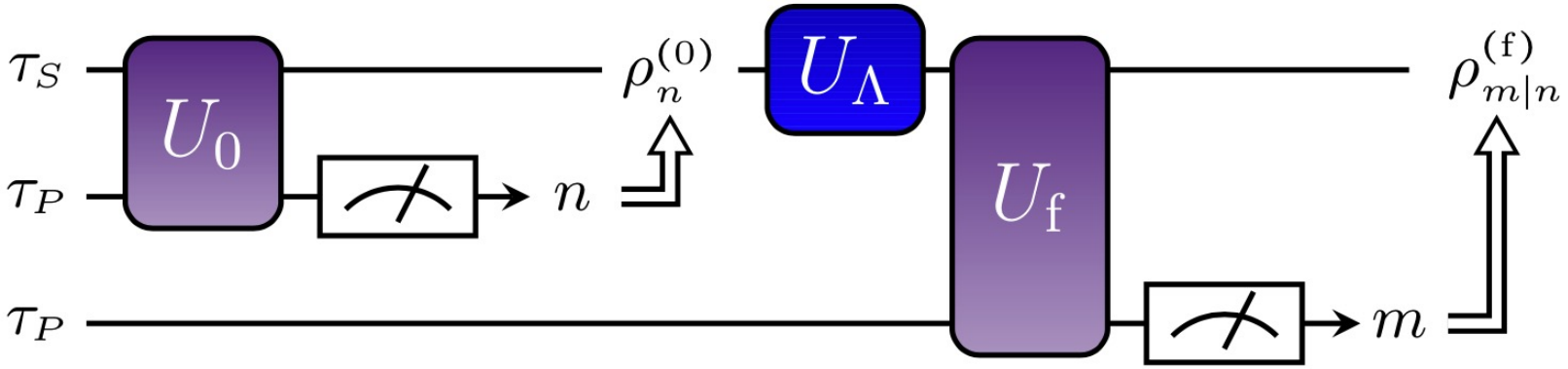
Work that is performed on, or extracted from the system can be estimated via a TPM scheme. Obtaining outcomes n and m implies the system is in those pure states.

$$|E_n^{(0)}\rangle \qquad\qquad |E_m^{(\mathrm{f})}\rangle$$

One can associate a transition probability and a work value to each transition. Over all possible transitions the work value is given by:

$$\begin{aligned}\langle W\rangle_\Lambda &= \sum_{m,n} p_n^{(0)} p_{n\to m}\left(E_m^{(\mathrm{f})} - E_n^{(0)}\right)\\ &= \mathrm{Tr}\left(H^{(\mathrm{f})}\rho^{(\mathrm{f})}\right) - \mathrm{Tr}\left(H^{(0)}\tau^{(0)}\right)\\ &=: \Delta E_\Lambda\end{aligned}$$

Two-point measurement (TPM) scheme (**non-ideal** case)



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$$\rho_n^{(0)} = \sum_l q_{ll|n}^{(0)} \left|E_l^{(0)}\right\rangle \left\langle E_l^{(0)}\right| \qquad \rho_{m|n}^{(f)}$$

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Work that is performed on, or extracted from the system can be estimated via a TPM scheme. Obtaining outcomes n and m implies the system is in those pure states.

$$\rho_n^{(0)} = \sum_l q_{ll|n}^{(0)} \left|E_l^{(0)}\right\rangle \left\langle E_l^{(0)}\right| \qquad \rho_{m|n}^{(f)}$$

One can associate a transition probability and a work value to each transition. Over all possible transitions the work value is given by:

$$\langle W \rangle_{\mathrm{non-id}} = \sum_{m,n} p_n^{(0)} p(m|n) \left(E_m^{(\mathrm{f})} - E_n^{(0)}\right)$$

Two-point measurement (TPM) scheme (**non-ideal** case)

$$(\tau^{(0)}, H^{(0)}) \xrightarrow{\Lambda} (\rho^{(\mathrm{f})}, H^{(\mathrm{f})})$$

Work that is performed on, or extracted from the system can be estimated via a TPM scheme. Obtaining outcomes n and m implies the system is in those pure states.

$$\rho_n^{(0)} = \sum_l q_{ll|n}^{(0)} \, |E_l^{(0)}\rangle \langle E_l^{(0)}| \qquad \rho_{m|n}^{(f)}$$

One can associate a transition probability and a work value to each transition. Over all possible transitions the work value is given by:

$$\langle W \rangle_{\mathrm{non-id}} = \sum_{m,n} p_n^{(0)} p(m|n) \left(E_m^{(\mathrm{f})} - E_n^{(0)}\right)$$

Corrected expression for work

$$\langle W \rangle_{\text{non-id}} = \sum_{m,n} p_n^{(0)} p(m|n) \left(E_m^{(\text{f})} - E_n^{(0)} \right)$$

$$= C_{\max} \langle W \rangle_{\Lambda} + \sum_{\substack{m,n \\ l \neq n}} p_{l \to m} \, q_{ll|n}^{(0)} \, p_n^{(0)} \left(E_m^{(\text{f})} - E_n^{(0)} \right)$$

Corrected expression for work

$$\langle W \rangle_{\text{non-id}} = \sum_{m,n} p_n^{(0)} p(m|n) \left(E_m^{(\text{f})} - E_n^{(0)} \right)$$

$$= C_{\max} \langle W \rangle_{\Lambda} + \sum_{\substack{m,n \\ l \neq n}} p_{l \to m} \, q_{ll|n}^{(0)} \, p_n^{(0)} \left(E_m^{(\text{f})} - E_n^{(0)} \right)$$

trustworthiness

ideal case

Positive or negative

Corrected expression for work

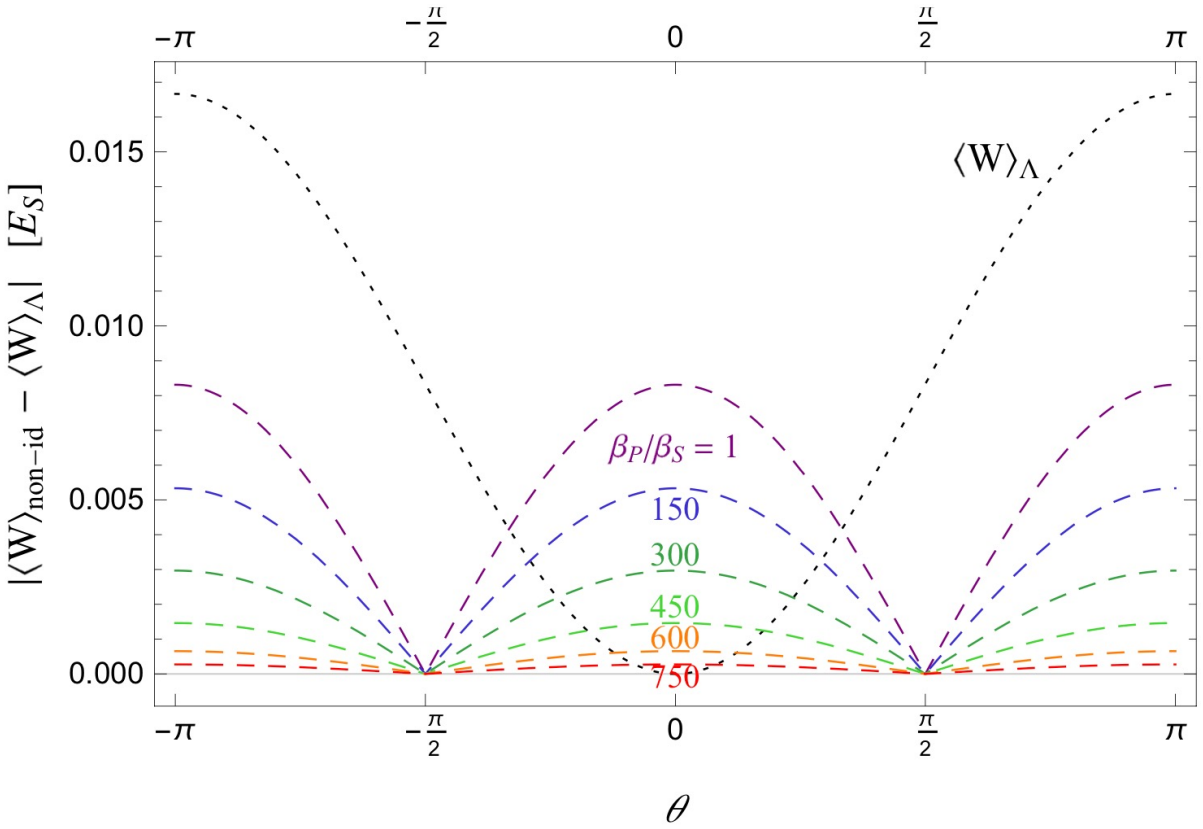
$$\langle W \rangle_{\text{non-id}} = \sum_{m,n} p_n^{(0)} p(m|n) \left(E_m^{(\text{f})} - E_n^{(0)} \right)$$

$$= C_{\max} \langle W \rangle_{\Lambda} + \sum_{\substack{m,n \\ l \neq n}} p_{l \to m} \, q_{ll|n}^{(0)} \, p_n^{(0)} \left(E_m^{(\text{f})} - E_n^{(0)} \right)$$

$$|\langle W \rangle_{\text{non-id}} - \langle W \rangle_{\Lambda}| \leq (1 - C_{\max}) ||H^{(\text{f})}||_{\infty}$$

Difference to the idea case is non-negligible, but can be bounded by the Hamiltonian of the system.

Corrected expression for work



Difference up to twice as much